

Algebra – GCSE Mathematics

1. No.8

Solve $7x - 22 = 4x + 29$

[3 marks]

$$7x - 4x = 29 + 22$$

$$\frac{3x}{3} = \frac{51}{3}$$

$$x = \underline{\underline{17}}$$

$$\begin{array}{r} 17 \\ 3 \overline{) 51} \\ \underline{3} \\ 21 \\ \underline{21} \\ 0 \end{array}$$

$$x = \underline{\quad 17 \quad}$$

2. No.17

Rearrange $y = \frac{3x+7}{x}$ to make x the subject.

[4 marks]

$$y = \frac{3x+7}{x} \quad (\text{Multiply both sides by } x.)$$

$$yx = 3x + 7$$

$$yx - 3x = 7$$

$$x(\cancel{y-3}) = 7$$

$$\cancel{y-3} \quad y-3$$

$$x = \frac{7}{y-3}$$

Answer $x = \frac{7}{y-3}$

3. No.19

 A , B and C are numbers.Here is some information about B and C .

B	$\frac{7}{4}$ of A
C	A increased by 150%

Work out C as a fraction of B .

$$C = A \times 1.5$$

[4 marks]

$$C = \underline{\underline{1.5A}}$$

 $B = \frac{7}{4} \text{ of } A$
Multiply both sides by 4.

$$B = \frac{7}{4} \times A$$

$$\frac{C}{B} = \frac{\frac{7}{4}A}{\frac{3}{2}A}$$

$$\frac{4B}{4} = \frac{7A}{4}$$

$$B = \frac{7A}{4}$$

$$C = \text{Increased by } 150\% \\ = \frac{150}{100} = 1.5A$$

$$\frac{7}{4} = \frac{7}{4} \div \frac{3}{2}$$

$$\frac{3}{2} = \frac{7}{4} \times \frac{2}{3}$$

$$\frac{C}{B} = \frac{14}{12} \left(\frac{7}{6} \right)$$

Answer $\frac{7}{6}$

4. No.20

$$5x^3 + ax^2 + bx + c \equiv kx^3 + (2-k)x^2 + (a^2-1)x + \frac{b}{2}$$

Work out the values of a , b and c .

[3 marks]

$$5x^3 + ax^2 + bx + c = kx^3 + (2-k)x^2 + (a^2-1)x + \frac{b}{2}$$

$5x^3 = kx^3$	$bx = (a^2-1)x$	$c = \frac{b}{2}$
$k = \underline{\underline{5}}$	$b = (-3)^2 - 1$	$c = 8/2$
$ax^2 = (2-k)x^2$	$= 9 - 1$	$c = \underline{\underline{4}}$
$a = 2 - 5$	$= \underline{\underline{8}}$	
$a = \underline{\underline{-3}}$		

$$a = \underline{\underline{-3}} \quad b = \underline{\underline{8}} \quad c = \underline{\underline{4}}$$

5. No.22

$$x_{n+1} = 5 - \frac{1}{x_n}$$

Use $x_1 = 1$ to work out an approximate solution to $x = 5 - \frac{1}{x}$

Give your answer to 4 significant figures.

[3 marks]

Let $n=1$

$$x_2 = 5 - \frac{1}{x_1}$$

$$= 5 - \frac{1}{1}$$

$$= \underline{4}$$

$$x_3 = 5 - \frac{1}{x_2}$$

$$= 5 - \frac{1}{2}$$

$$= 5 - \frac{1}{4}$$

$$= \underline{\underline{4.75}}$$

$$x_4 = 5 - \frac{1}{x_3}$$

$$= 5 - \frac{1}{8}$$

$$= 4.78947$$

$$x_5 = 4.79121$$

$$x_6 = 4.79128$$

$$x_7 = 4.79129$$

$$x = \underline{\underline{4.791}}$$

$x =$

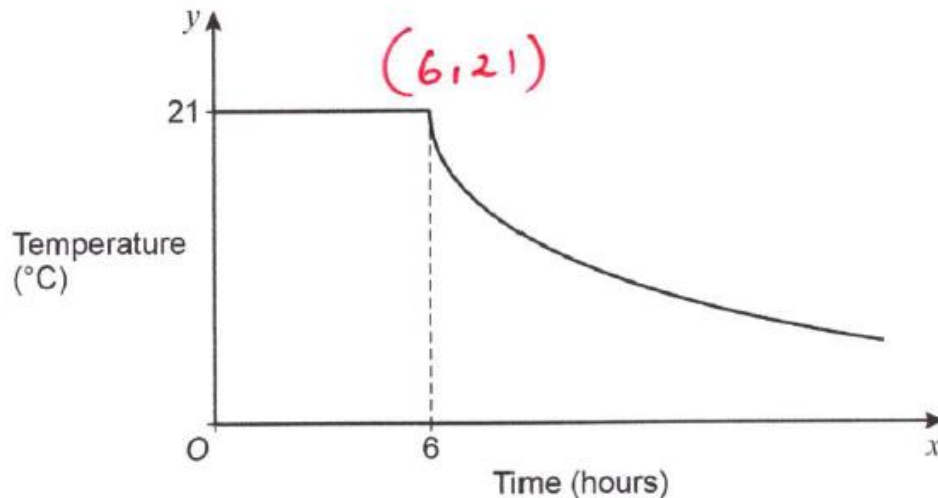
4.791

6. No.24

A room is kept at a constant temperature of 21°C for 6 hours.

The heating is then turned off and the room begins to cool.

Here is a sketch graph showing the temperature, $y^{\circ}\text{C}$, of the room at time x hours.



- (a) Assume the equation of the curved part is $y = \frac{k}{x}$ where k is a constant.

Work out the value of y when $x = 12$

[2 marks]

When $x=6$, $y=21$

Substitute values to get k . When $x=12$

$$21 = \frac{k}{6}$$

$$k = 21 \times 6$$

$$k = \underline{\underline{126}}$$

$$y = \frac{126}{x}$$

$$y = \frac{126}{12}$$

$$y = \underline{\underline{10.5}}$$

$y = 10.5$

(b) In fact,

the equation of the curved part is $y = A \times \left(\frac{1}{3}\right)^{\frac{1}{6}x}$ where A is a **different** constant.

How does this affect the value of y when $x = 12$?

(6, 21)

Tick **one** box.

You **must** show working to support your answer.

[2 marks]

☐

The value of y is greater than the answer to part (a).

☒

The value of y is less than the answer to part (a).

☐

The value of y is the same as the answer to part (a).

$$\text{When } x = 6, y = 21$$

$$y = A \left(\frac{1}{3}\right)^{\frac{1}{6}x}$$

$$A = \frac{21}{\left(\frac{1}{3}\right)^{\frac{1}{6} \times 6}}$$

$$\left(\frac{1}{3}\right)^{\frac{1}{6} \times 6}$$

$$A = \frac{21}{\frac{1}{3}}$$

$$A = 21 \times \frac{3}{1}$$

$$A = \underline{\underline{63}}$$

$$\text{When } x = 12$$

$$y = 63 \times \left(\frac{1}{3}\right)^{\frac{1}{6} \times 12}$$

$$y = 63 \times \left(\frac{1}{3}\right)^2$$

$$y = 63 \times \frac{1}{9}$$

$$y = \underline{\underline{7}}$$

The value is less than in part a

7. No.17

(a) a and b are different prime numbers.

$$a^3 \times b^2 = 200$$

Work out the value of $a^4 \times b$

[3 marks]

$$\begin{array}{l}
 200 \\
 \wedge \\
 2 \quad 100 \\
 \wedge \\
 2 \quad 50 \\
 \wedge \\
 2 \quad 25 \\
 \wedge \\
 5 \quad 5
 \end{array}
 \qquad
 \begin{array}{l}
 a^3 \times b^2 \\
 = 2^3 \times 5^2 \\
 = a^4 \times b \\
 = 2^4 \times 5 \\
 = \underline{\underline{80}}
 \end{array}$$

Answer 80.(b) c and d are different prime numbers.Circle the equation for which $c^4 \times d^2 \times e$ is a cube number.

[1 mark]

$e = cd$

$e = c^2d$

$e = c^2d^2$

$e = c^3d^3$

$$\begin{aligned}
 & (c^4 \times d^2 \times e) \quad (c^2d) \\
 & = c^4 \times c^2 \cdot d^2 \times d \quad (\text{use law of indices}) \\
 & = c^6 d^3 \quad (\text{it can be expressed as } \underline{\underline{(c^2d)^3}})
 \end{aligned}$$

8. No.19

$$f(x) = x - 3 \quad g(x) = 4x - 7$$

(a) Work out the value of $fg(6)$

fg (Means do g first then followed by f) [2 marks]

$$\begin{array}{l|l} g(6) = 4(6) - 7 & f(14) = x - 3 \\ = 24 - 7 & = 14 - 3 \\ = \underline{17} & = \underline{11} \end{array}$$

Answer 11

(b) Solve $(f(x))^2 = g(x)$

[4 marks]

$$(x-3)^2 = (4x-7)$$

$$(x-3)(x-3) = 4x-7$$

$$x^2 - 3x - 3x + 9 = 4x - 7$$

$$x^2 - 6x + 9 = 4x - 7$$

$$x^2 - 6x - 4x + 9 + 7 = 0$$

$$x^2 - 10x + 16 = 0$$

$$p = 16 \quad (-8, -2)$$

$$q = -10$$

$$(x-8)(x-2) = 0$$

$$x-8=0$$

$$x = \underline{8}$$

$$x-2=0$$

$$x = \underline{2}$$

Answer $x=2$ or $x=8$