Polar coordinates – A2 Further Mathematics P1

1. June/2022/Paper_7367/01/No.9

Roberto is solving this mathematics problem:

> The curve C_1 has polar equation
>
> $$r^2 = 9\sin 2\theta$$
>
> for all possible values of θ
>
> Find the area enclosed by C_1

Roberto's solution is as follows:

$$A = \frac{1}{2}\int_{-\pi}^{\pi} 9\sin 2\theta \, \mathrm{d}\theta$$

$$= \left[-\frac{9}{4}\cos 2\theta\right]_{-\pi}^{\pi}$$

$$= 0$$

(a) Sketch the curve C_1

[2 marks]

(b) Explain what Roberto has done wrong.

[2 marks]

(c) Find the area enclosed by C_1

[2 marks]

(d) P and Q are distinct points on C_1 for which r is a maximum.
P is above the initial line.

Find the polar coordinates of P and Q

[2 marks]

(e) The matrix $\mathbf{M} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ represents the transformation T

T maps C_1 onto a curve C_2

(e) (i) T maps P onto the point P'

Find the polar coordinates of P'

[4 marks]

(e) (ii) Find the area enclosed by C_2

Fully justify your answer.

[2 marks]