## Matrices - A2 Further Mathematics P2

1. June/2022/Paper_7367/02/No. 11
(a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$
\mathbf{M}=\left[\begin{array}{cc}
\frac{5}{2} & -\frac{3}{2} \\
-\frac{3}{2} & \frac{13}{2}
\end{array}\right]
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(b) (i) Describe how the directions of the invariant lines of the transformation represented by $\mathbf{M}$ are related to each other.

Fully justify your answer.
[2 marks]
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(b) (ii) Describe fully the transformation represented by $\mathbf{M}$
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2. June/2022/Paper_7367/02/No. 13
(a) The matrix $\mathbf{A}$ represents a reflection in the line $y=m x$, where $m$ is a constant.

Show that $\mathbf{A}=\left(\frac{1}{m^{2}+1}\right)\left[\begin{array}{cc}1-m^{2} & 2 m \\ 2 m & m^{2}-1\end{array}\right]$
You may use the result in the formulae booklet.
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(b) The matrix $\mathbf{B}$ is defined as $\mathbf{B}=\left[\begin{array}{ll}3 & 0 \\ 0 & 3\end{array}\right]$

Show that $(\mathrm{BA})^{2}=k \mathbf{I}$
where $\mathbf{I}$ is the $2 \times 2$ identity matrix and $k$ is an integer.
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(c) (i) The diagram below shows a point $P$ and the line $y=m x$

Draw four lines on the diagram to demonstrate the result proved in part (b).
Label as $P^{\prime}$ the image of $P$ under the transformation represented by (BA) ${ }^{2}$

(c) (ii) Explain how your completed diagram shows the result proved in part (b).
[2 marks]
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(d) The matrix $\mathbf{C}$ is defined as $\mathbf{C}=\left[\begin{array}{cc}\frac{12}{5} & \frac{9}{5} \\ \frac{9}{5} & -\frac{12}{5}\end{array}\right]$

Find the value of $m$ such that $\mathbf{C}=\mathbf{B A}$
Fully justify your answer.
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