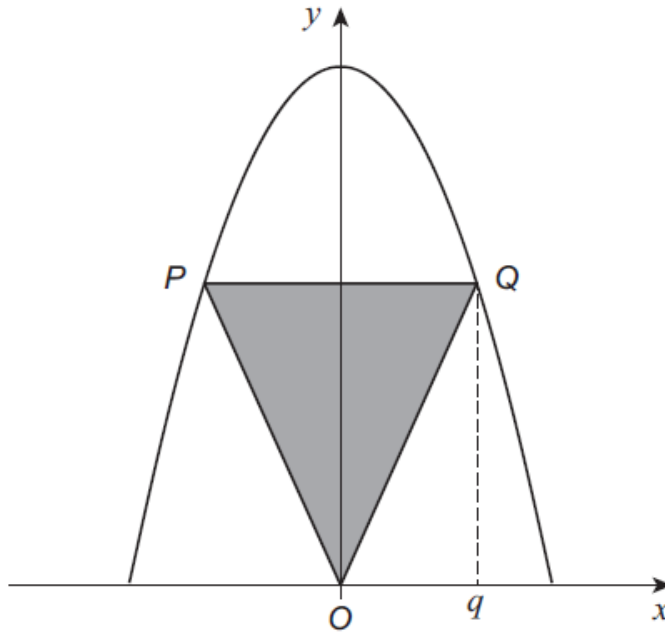**Integration – A2 Mathematics P2**

1. June/2022/Paper_7357/02/No.7

The curve $y = 15 - x^2$ and the isosceles triangle $OPQ$ are shown on the diagram below.



Vertices $P$ and $Q$ lie on the curve such that $Q$ lies vertically above some point $(q, 0)$

The line $PQ$ is parallel to the $x$-axis.

(a) Show that the area, $A$, of the triangle $OPQ$ is given by

$$A = 15q - q^3 \quad \text{for } 0 < q < c$$

where $c$ is a constant to be found.

[3 marks]

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**(b)**     Find the exact maximum area of triangle *OPQ*.

Fully justify your answer.

**[6 marks]**

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**2.** June/2022/Paper_7357/02/No.10(b)

**(b)**   A refined model assumes the rate of increase of the number of plants damaged by insects is given by

$$\frac{dx}{dt} = \frac{x(900 - x)}{2700}$$

**(b) (i)**   Show that

$$\int \left( \frac{A}{x} + \frac{B}{900 - x} \right) dx = \int dt$$

where $A$ and $B$ are positive integers to be found.

**[3 marks]**

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**2.** June/2022/Paper_7357/02/No.10(b)

**(b) (ii)** Hence, find $t$ in terms of $x$.

**[5 marks]**

**(b) (iii)** Hence, find the number of days it takes from when the damage is first noticed until half of the plants are damaged by the insects.

**[2 marks]**