Further algebra and functions – AS Further Mathematics P1

1. June/2022/Paper_7366/01/No.2

   The quadratic equation $x^2 + px + q = 0$ has roots α and β

   Which of the following is equal to $\alpha\beta$?

   Circle your answer.

   [1 mark]

   p $\qquad\qquad$ $-p$ $\qquad\qquad$ q $\qquad\qquad$ $-q$

2. June/2022/Paper_7366/01/No.9

   (a) Show that, for $r > 0$,

   $$\ln(r+2) - \ln r = \ln\left(1 + \frac{2}{r}\right)$$

   [1 mark]

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(b) Hence, using the method of differences, show that

$$\sum_{r=1}^{n} \ln\left(1 + \frac{2}{r}\right) = \ln\left(\frac{1}{2}(n+a)(n+b)\right)$$

where a and b are integers to be found.

[4 marks]

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3. June/2022/Paper_7366/01/No.13

A curve C_1 has equation

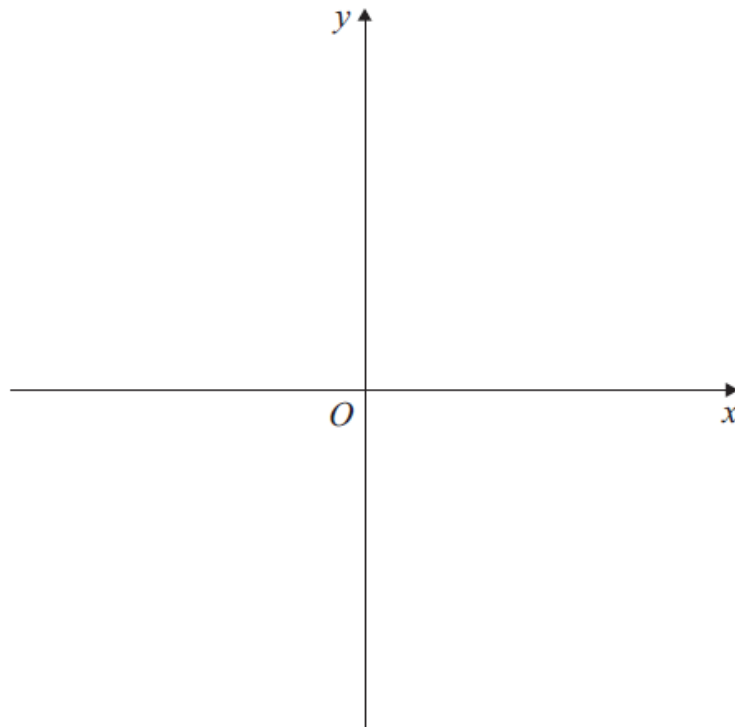$$y = \frac{2x + 7}{3x + 5}$$

(a) Write down the equations of the asymptotes of curve C_1

[2 marks]

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(b) On the axes below, sketch the graph of curve C_1

Indicate the values of the intercepts of the curve with the axes.

[3 marks]

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(c) Hence, or otherwise, solve the inequality

$$\frac{2x+7}{3x+5} \geq 0$$

[2 marks]

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(d) Curve C_2 is a reflection of curve C_1 in the line $y = -x$

Find an equation for curve C_2 in the form $y = f(x)$

[3 marks]

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The function f is defined by

$$f(x) = \frac{x^2 - 3}{x^2 + px + 7} \qquad x \in \mathbb{R}$$

where p is a constant.

The graph of $y = f(x)$ has only one asymptote.

(a) Write down the equation of the asymptote.

[1 mark]

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(b) Find the set of possible values of p

[4 marks]

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(c)   Find the coordinates of the points at which the graph of $y = f(x)$ intersects the axes.

[3 marks]

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(d)   A curve C has equation

$$y = \frac{x^2 - 3}{x^2 - 3x + 7}$$

The curve C has a local minimum at the point M as shown in the diagram.
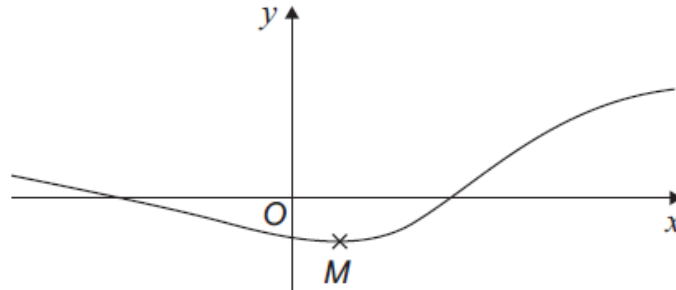
The line $y = k$ intersects curve C

(d) (i)   Show that

$$19k^2 - 16k - 12 \leq 0$$

[5 marks]

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(d) (ii) Hence, find the y-coordinate of point M

[2 marks]

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