Differential equations – A2 Further Mathematics P2

1. June/2022/Paper_7367/02/No.9(b_ c)

A curve passes through the point (5, 12.3) and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (x^2 - 9)^{\frac{1}{2}} + \frac{2xy}{x^2 - 9} \qquad x > 3$$

Use Euler's step by step method once, and then the midpoint formula

$$y_{r+1} = y_{r-1} + 2h\mathrm{f}(x_r, y_r), \quad x_{r+1} = x_r + h$$

(b) (i) Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (x^2 - 9)^{\frac{1}{2}} + \frac{2xy}{x^2 - 9} \qquad (x > 3)$$

[6 marks]

(b) (ii) Given that y satisfies the differential equation in part **(b)(i)** and that $y = 12.3$ when $x = 5$, find the value of y when $x = 5.2$

Give your answer to six significant figures.

[3 marks]

(c) Comment on the accuracy of your answer to part **(a)**.

[1 mark]

2. June/2022/Paper_7367/02/No.14

On an isolated island some rabbits have been accidently introduced.

In order to eliminate them, conservationists have introduced some birds of prey.

At time t years $(t \geq 0)$ there are x rabbits and y birds of prey.

At time $t = 0$ there are 1755 rabbits and 30 birds of prey.

When $t > 0$ it is assumed that:

- the rabbits will reproduce at a rate of $a\%$ per year
- each bird of prey will kill, on average, b rabbits per year
- the death rate of the birds of prey is c birds per year
- the number of birds of prey will increase at a rate of $d\%$ of the rabbit population per year.

This system is represented by the coupled differential equations:

$$\frac{dx}{dt} = 0.4x - 13y \qquad (1)$$

$$\frac{dy}{dt} = 0.01x - 1.95 \qquad (2)$$

(a) State the value of a, the value of b, the value of c and the value of d

[2 marks]

(b) Solve the coupled differential equations to find both x and y in terms of t

[9 marks]

(c) Given that x and y are both positive for $0 \leq t \leq 5$, use your answer to part **(b)** to show that the conservationists' plan will succeed.

[3 marks]