Differential equations – A2 Further Mathematics P1

1. June/2022/Paper_7367/01/No.1

The displacement of a particle from its equilibrium position is x metres at time t seconds.

The motion of the particle obeys the differential equation

$$\frac{d^2x}{dt^2} = -9x$$

Calculate the period of its motion in seconds.

Circle your answer.

[1 mark]

$\frac{\pi}{9}$ $\frac{2\pi}{9}$ $\frac{\pi}{3}$ $\frac{2\pi}{3}$

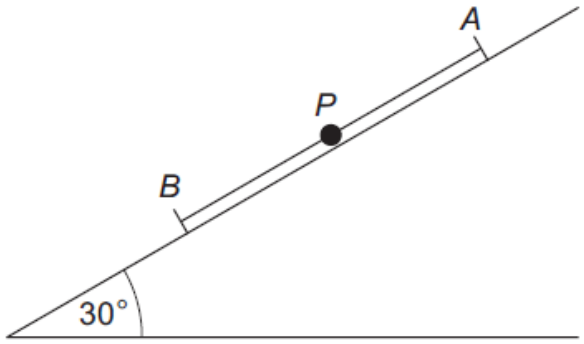
2. June/2022/Paper_7367/01/No.11

In this question use g as $10\,\text{m}\,\text{s}^{-2}$

A smooth plane is inclined at 30° to the horizontal.
The fixed points A and B are 3.6 metres apart on the line of greatest slope of the plane, with A higher than B

A particle P of mass 0.32 kg is attached to one end of each of two light elastic strings. The other ends of these strings are attached to the points A and B respectively.

The particle P moves on a straight line that passes through A and B



The natural length of the string AP is 1.4 metres.
When the extension of the string AP is e_A metres, the tension in the string AP is $7e_A$ newtons.
The natural length of the string BP is 1 metre.
When the extension of the string BP is e_B metres, the tension in the string BP is $9e_B$ newtons.

The particle P is held at the point between A and B which is 0.2 metres from its equilibrium position and lower than its equilibrium position.
The particle P is then released from rest.

At time t seconds after P is released, its displacement towards B from its equilibrium position is x metres.

(a) Show that during the subsequent motion the object satisfies the equation

$$\ddot{x} + 50x = 0$$

Fully justify your answer.

[5 marks]

(b) The experiment is repeated in a large tank of oil.
During the motion the oil causes a resistive force of kv newtons to act on the particle, where $v\,\text{m s}^{-1}$ is the speed of the particle.

The oil causes critical damping to occur.

(b) (i) Show that $k = \dfrac{16\sqrt{2}}{5}$

[3 marks]

(b) (ii) Find x in terms of t, giving your answer in exact form.

[6 marks]

(b) (iii) Calculate the maximum speed of the particle.

[5 marks]