Complex numbers – A2 Further Mathematics P1

1. June/2022/Paper_7367/01/No.2

Simplify

$$\frac{\cos\left(\frac{6\pi}{13}\right)+\mathrm{i}\sin\left(\frac{6\pi}{13}\right)}{\cos\left(\frac{2\pi}{13}\right)-\mathrm{i}\sin\left(\frac{2\pi}{13}\right)}$$

Tick (✓) **one** box.

[1 mark]

$\cos\left(\frac{8\pi}{13}\right)+\mathrm{i}\sin\left(\frac{8\pi}{13}\right)$ ☐

$\cos\left(\frac{8\pi}{13}\right)-\mathrm{i}\sin\left(\frac{8\pi}{13}\right)$ ☐

$\cos\left(\frac{4\pi}{13}\right)+\mathrm{i}\sin\left(\frac{4\pi}{13}\right)$ ☐

$\cos\left(\frac{4\pi}{13}\right)-\mathrm{i}\sin\left(\frac{4\pi}{13}\right)$ ☐

2. June/2022/Paper_7367/01/No.5(a)

It is given that $z = -\frac{3}{2} + \mathrm{i}\frac{\sqrt{11}}{2}$ is a root of the equation

$$z^4 - 3z^3 - 5z^2 + kz + 40 = 0$$

where k is a real number.

(a) Find the other three roots.

[5 marks]

3. June/2022/Paper_7367/01/No.8

(a) The complex number w is such that

$$\arg(w + 2\text{i}) = \tan^{-1}\frac{1}{2}$$

It is given that $w = x + \text{i}y$, where x and y are real and $x > 0$

Find an equation for y in terms of x

[2 marks]

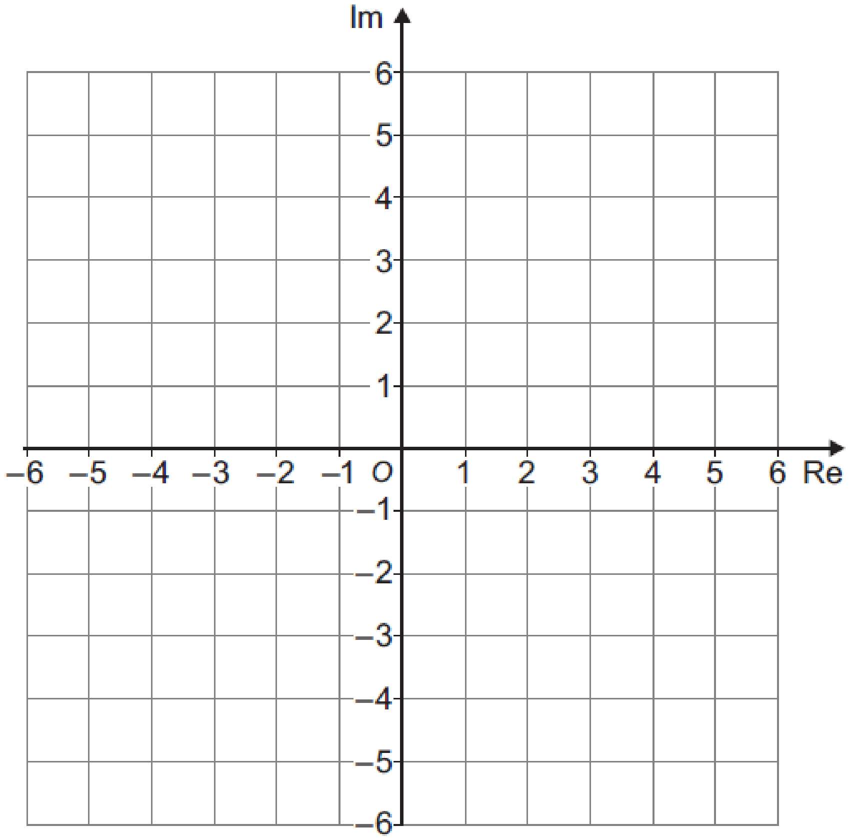
(b) The complex number z satisfies both

$$-\frac{\pi}{2} \leq \arg(z + 2\mathrm{i}) \leq \tan^{-1}\frac{1}{2} \qquad \textbf{and} \qquad |z - 2 + 3\mathrm{i}| \leq 2$$

The region R is the locus of z

Sketch the region R on the Argand diagram below.

[4 marks]



(c) z_1 is the point in R at which $|z|$ is minimum.

(c) (i) Calculate the exact value of $|z_1|$

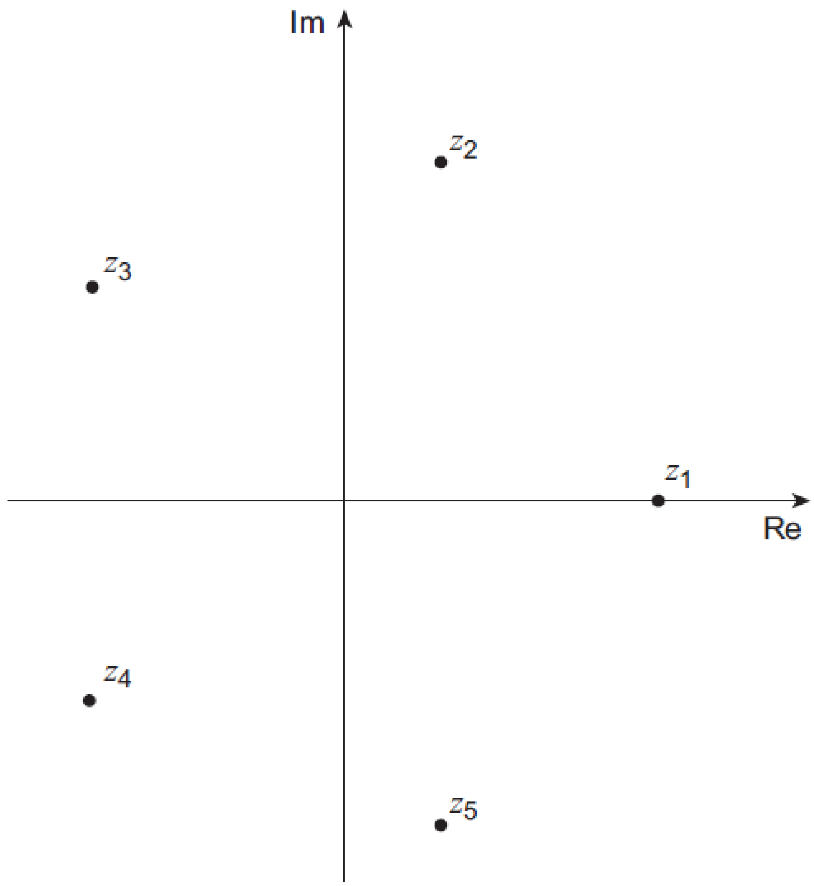
[3 marks]

(c) (ii) Express z_1 in the form $a + \mathrm{i}b$, where a and b are real.

[2 marks]

4. June/2022/Paper_7367/01/No.12

The Argand diagram shows the solutions to the equation $z^5 = 1$



(a) Solve the equation

$$z^5 = 1$$

giving your answers in the form $z = \cos\theta + i\sin\theta$, where $0 \leq \theta < 2\pi$

[2 marks]

(b) Explain why the points on an Argand diagram which represent the solutions found in part **(a)** are the vertices of a regular pentagon.

[2 marks]

(c) Show that if $c = \cos\theta$, where $z = \cos\theta + \mathrm{i}\sin\theta$ is a solution to the equation $z^5 = 1$, then c satisfies the equation

$$16c^5 - 20c^3 + 5c - 1 = 0$$

[5 marks]