AQA – Proof – A2 Mathematics P2

1. June/2021/Paper_7357/2/No.8

Kai is proving that $n^3 - n$ is a multiple of 3 for all positive integer values of n.

Kai begins a proof by exhaustion.

Step 1		$n^3 - n = n(n^2 - 1)$
Step 2	When $n = 3m$, where m is a non-negative integer	$n^3 - n = 3m(9m^2 - 1)$ which is a multiple of 3
Step 3	When $n = 3m + 1$,	$n^3 - n = (3m + 1)((3m + 1)^2 - 1)$
Step 4		$= (3m + 1)(9m^2)$ $= 3(3m + 1)(3m^2)$ which is a multiple of 3
Step 5	Therefore $n^3 - n$ is a multiple of 3 for all positive integer values of n	

(a) Explain the two mistakes that Kai has made after Step 3.

[2 marks]

(b) Correct Kai's argument from Step 4 onwards.

[4 marks]