AQA - Further calculus - A2 Further Mathematics P1

1. June/2021/Paper_7367/1/No.9

Use l'Hôpital's rule to show that

$$\lim_{x\to\infty}(x\mathrm{e}^{-x})=0$$

Fully justify your answer.

[4 marks]

2. June/2021/Paper_7367/1/No.10

Evaluate the improper integral

$$\int_0^8 \ln x \ \mathrm{d}x$$

showing the limiting process.

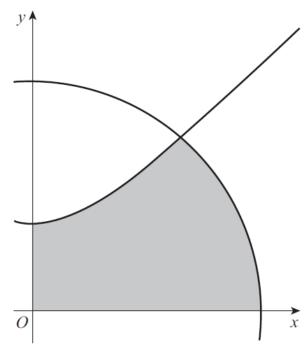
[6 marks]

3. June/2021/Paper_7367/1/No.14

The hyperbola *H* has equation $y^2 - x^2 = 16$

The circle *C* has equation $x^2 + y^2 = 32$

The diagram below shows part of the graph of *H* and part of the graph of *C*.



Show that the shaded region in the first quadrant enclosed by H, C, the x-axis and the y-axis has area

$$\frac{16\pi}{3} + 8 \ln \left(\frac{\sqrt{2} + \sqrt{6}}{2} \right)$$

[12 marks]