AQA – Further algebra and functions – AS Further Mathematics P1

1. June/2021/Paper_7366/1/No.2

Given that $f(x) = 3x - 1$ find the mean value of $f(x)$ over the interval $4 \leq x \leq 8$

Circle your answer.

[1 mark]

6 11 17 23

2. June/2021/Paper_7366/1/No.7

Show that the Maclaurin series for $\ln(e + 2ex)$ is

$$1 + 2x - 2x^2 + ax^3 - ...$$

where a is to be determined.

[3 marks]

3. June/2021/Paper_7366/1/No.9

(a) Use the standard formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} r(r+3) = an(n+1)(n+b)$$

where a and b are constants to be determined.

[4 marks]

(b) Hence, or otherwise, find a fully factorised expression for

$$\sum_{r=n+1}^{5n} r(r+3)$$

[3 marks]

4. June/2021/Paper_7366/1/No.11

(a) Show that, for all positive integers r,

$$\frac{1}{(r-1)!} - \frac{1}{r!} = \frac{r-1}{r!}$$

[1 mark]

(b) Hence, using the method of differences, show that

$$\sum_{r=1}^{n} \frac{r-1}{r!} = a + \frac{b}{n!}$$

where a and b are integers to be determined.

[3 marks]

5. June/2021/Paper_7366/1/No.12

The equation $x^3 - 2x^2 - x + 2 = 0$ has three roots. One of the roots is 2

(a) Find the other two roots of the equation.

[1 mark]

(b) Hence, or otherwise, solve

$$\cosh^3 \theta - 2\cosh^2 \theta - \cosh \theta + 2 = 0$$

giving your answers in an exact form.

[4 marks]

6. June/2021/Paper_7366/1/No.14

Curve C_1 has equation

$$\frac{x^2}{16}+\frac{y^2}{4}=1$$

(a) Curve C_2 is a reflection of C_1 in the line $y=x$

Write down an equation of C_2

[1 mark]

(b) Curve C_3 is a circle of radius 4, centred at the origin.

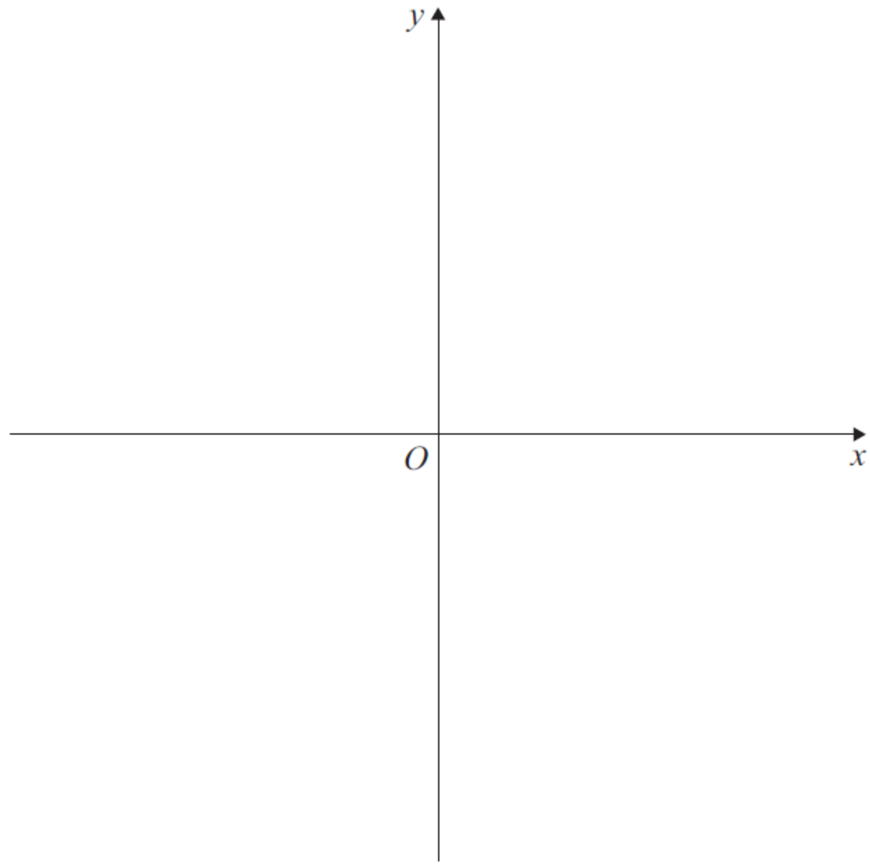
Describe a single transformation which maps C_1 onto C_3

[2 marks]

(c) Curve C_4 is a translation of C_1
The positive x-axis and the positive y-axis are tangents to C_4

(c) (i) Sketch the graphs of C_1 and C_4 on the axes opposite. Indicate the coordinates of the x and y intercepts on your graphs.

[2 marks]



(c) (ii) Determine the translation vector.

[2 marks]

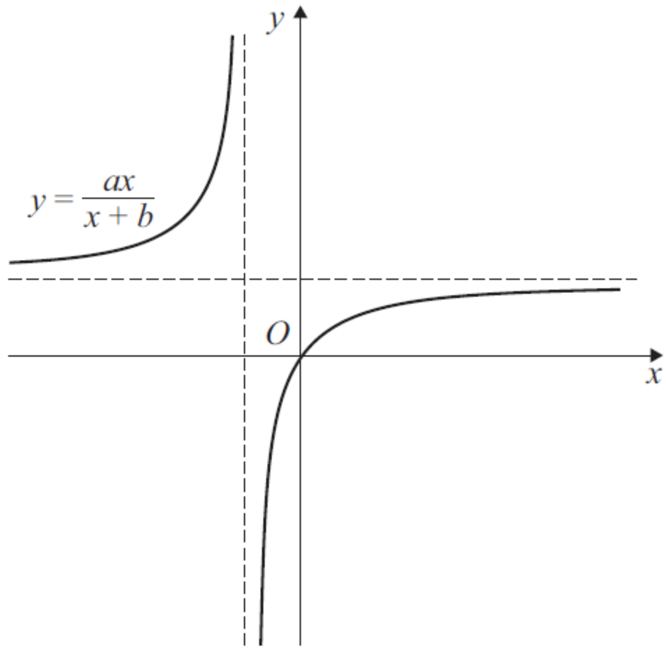
(c) (iii) The line $y = mx + c$ is a tangent to both C_1 and C_4
Find the value of m

[2 marks]

7. June/2021/Paper_7366/1/No.16

Curve C has equation $y = \dfrac{ax}{x+b}$ where a and b are constants.

The equations of the asymptotes to C are $x = -2$ and $y = 3$



(a) Write down the value of a and the value of b

[2 marks]

(b) The gradient of C at the origin is $\dfrac{3}{2}$

With reference to the graph, explain why there is exactly one root of the equation

$$\frac{ax}{x+b} = \frac{3x}{2}$$

[2 marks]

(c) Using the values found in part **(a)**, solve the inequality

$$\frac{ax}{x+b} \leq 1 - x$$

[4 marks]