AQA – Proof – AS Further Mathematics P1

1. June/2020/Paper_1/No.5

(a) Show that

$$r^2(r+1)^2 - (r-1)^2 r^2 = pr^3$$

where p is an integer to be found.

[1 mark]

(b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

[3 marks]

2. June/2020/Paper_1/No.7

Prove by induction that, for all integers $n \geq 1$, the expression $7^n - 3^n$ is divisible by 4

[4 marks]

3. June/2019/Paper_1/No.7

(a) Show that

$$\frac{1}{r-1}-\frac{1}{r+1}\equiv\frac{A}{r^2-1}$$

where A is a constant to be found.

[1 mark

(b) Hence use the method of differences to show that

$$\sum_{r=2}^{n}\frac{1}{r^2-1}\equiv\frac{an^2+bn+c}{4n(n+1)}$$

where a, b and c are integers to be found.

[4 marks

4. June/2019/Paper_1/No.12

The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

(a) Prove by induction that, for all integers $n \geq 1$,

$$\mathbf{A}^n = \begin{bmatrix} 1 & 3^n - 1 \\ 0 & 3^n \end{bmatrix}$$

[4 marks]

(b) Find all invariant lines under the transformation matrix **A**.

Fully justify your answer.

[6 marks]

(c) Find a line of invariant points under the transformation matrix **A**.

[2 marks]