AQA – Proof – A2 Mathematics P2

1.   June/2020/Paper_2/No.7

a and b are two positive irrational numbers.

The sum of a and b is rational.

The product of a and b is rational.

Caroline is trying to prove $\dfrac{1}{a}+\dfrac{1}{b}$ is rational.

Here is her proof:

Step 1	$\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{2}{a+b}$
Step 2	2 is rational and $a+b$ is non-zero and rational.
Step 3	Therefore $\dfrac{2}{a+b}$ is rational.
Step 4	Hence $\dfrac{1}{a}+\dfrac{1}{b}$ is rational.

(a) (i)   Identify Caroline's mistake.

[1 mark]

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(a) (ii)   Write down a correct version of the proof.

[2 marks]

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(b) Prove by contradiction that the difference of any rational number and any irrational number is irrational.

[4 marks]

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