AQA – Proof – A2 Further Mathematics P2

1. June/2020/Paper_2/No.10

The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 0 \qquad u_{n+1} = \frac{5}{6 - u_n}$$

Prove by induction that, for all integers $n \geq 1$,

$$u_n = \frac{5^n - 5}{5^n - 1}$$

[6 marks]

2. June/2019/Paper_2/No.10

Prove by induction that $f(n) = n^3 + 3n^2 + 8n$ is divisible by 6 for all integers $n \geq 1$

[7 marks]

3. June/2019/Paper_2/No.14

Let

$$S_n = \sum_{r=1}^{n} \frac{1}{(r+1)(r+3)}$$

where $n \geq 1$

(a) Use the method of differences to show that

$$S_n = \frac{5n^2 + an}{12(n+b)(n+c)}$$

where a, b and c are integers.

[6 marks]

(b) Show that, for any number k greater than $\frac{12}{5}$, if the difference between $\frac{5}{12}$ and S_n is less than $\frac{1}{k}$, then

$$n > \frac{k - 5 + \sqrt{k^2 + 1}}{2}$$

[6 marks]