AQA – Matrices – AS Further Mathematics P1

1. June/2020/Paper_1/No.4

The matrices **A** and **B** are such that

$$\mathbf{A} = \begin{bmatrix} 2 & a & 3 \\ 0 & -2 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -3 \\ -2 & 4a \\ 0 & 5 \end{bmatrix}$$

(a) Find the product **AB** in terms of a.

[2 marks]

(b) Find the determinant of **AB** in terms of a.

[1 mark]

(c) Show that **AB** is singular when $a = -1$

[2 marks]

2. June/2020/Paper_1/No.9

The quadratic equation $2x^2 + px + 3 = 0$ has two roots, α and β, where $\alpha > \beta$.

(a) (i) Write down the value of $\alpha\beta$.

[1 mark]

(a) (ii) Express $\alpha + \beta$ in terms of p.

[1 mark]

(b) Hence find $(\alpha - \beta)^2$ in terms of p.

[2 marks]

3. June/2020/Paper_1/No.10

(a) Show that the equation

$$y = \frac{3x - 5}{2x + 4}$$

can be written in the form

$$(x + a)(y + b) = c$$

where a, b and c are integers to be found.

[3 marks]

(b) Write down the equations of the asymptotes of the graph of

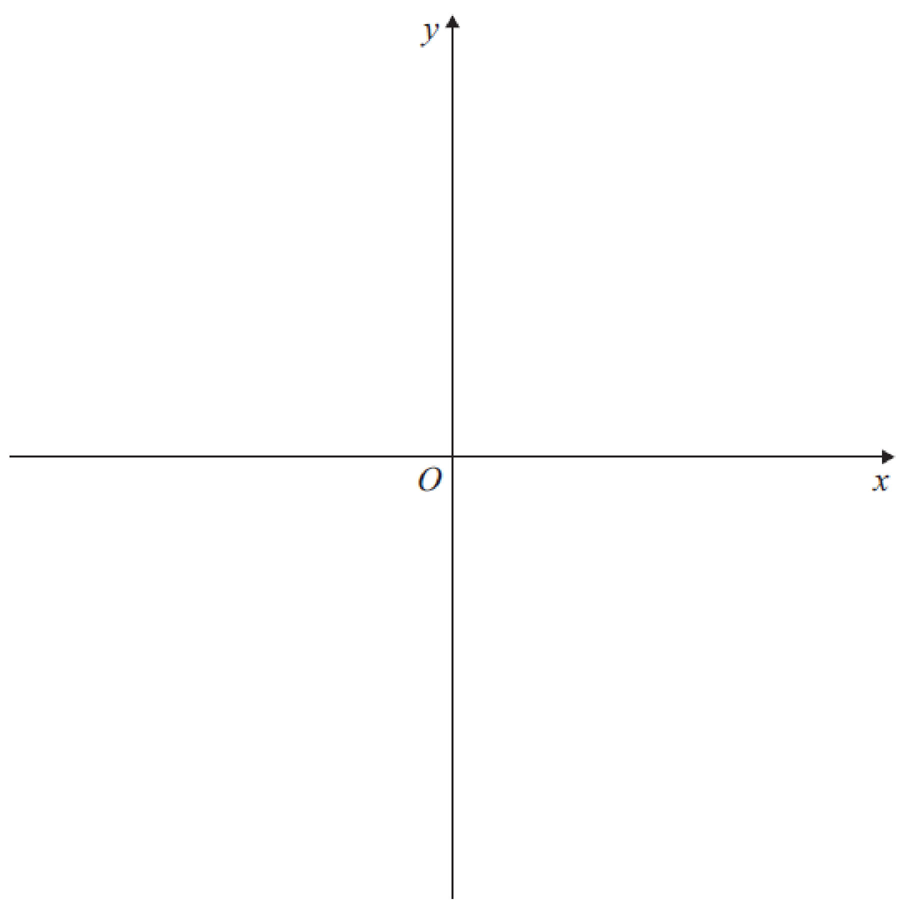
$$y = \frac{3x - 5}{2x + 4}$$

[2 marks]

(c) Sketch, on the axes provided, the graph of

$$y = \frac{3x - 5}{2x + 4}$$

[3 marks]



4. June/2020/Paper_1/No.14

(a) Given

$$\frac{x+7}{x+1} \leq x+1$$

show that

$$\frac{(x+a)(x+b)}{x+c} \geq 0$$

where a, b, and c are integers to be found.

[4 marks]

(b) Briefly explain why this statement is incorrect.

$$\frac{(x+p)(x+q)}{x+r} \geq 0 \Leftrightarrow (x+p)(x+q)(x+r) \geq 0$$

[1 mark]

5. June/2019/Paper_1/No.5

A hyperbola H has the equation

$$\frac{x^2}{a^2} - \frac{y^2}{4a^2} = 1$$

where a is a positive constant.

(a) Write down the equations of the asymptotes of H.

[1 mark]

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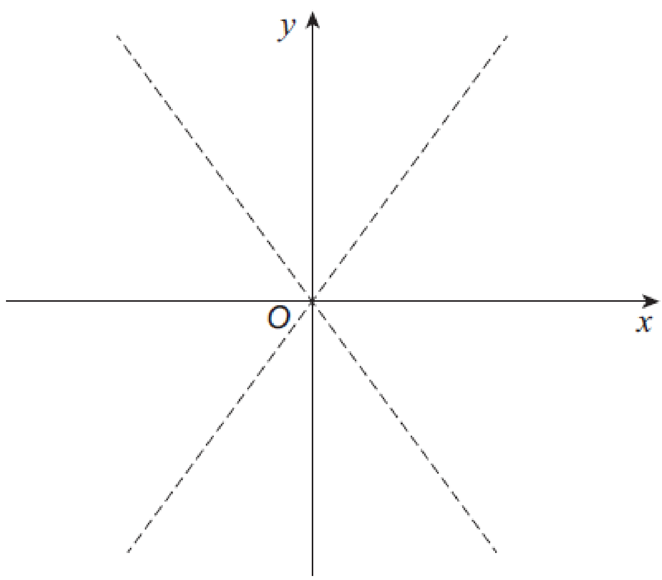
(b) Sketch the hyperbola H on the axes below, indicating the coordinates of any points of intersection with the coordinate axes.

The asymptotes have already been drawn.

[2 marks]



(c) The finite region bounded by *H*, the positive x-axis, the positive y-axis and the line $y = a$ is rotated through 360° about the y-axis.

Show that the volume of the solid generated is ma^3, where $m = 3.40$ correct to three significant figures.

[5 marks]

6. June/2019/Paper_1/No.10

(a) Using the definition of $\cosh x$ and the Maclaurin series expansion of e^x, find the first three non-zero terms in the Maclaurin series expansion of $\cosh x$.

[3 marks]

(b) Hence find a trigonometric function for which the first three terms of its Maclaurin series are the same as the first three terms of the Maclaurin series for $\cosh(ix)$.

[3 marks]

7. June/2019/Paper_1/No.11

(a) Curve C has equation

$$y = \frac{x^2 + px - q}{x^2 - r}$$

where p, q and r are positive constants.

Write down the equations of its asymptotes.

[2 marks]

(b) Find the set of possible y-coordinates for the graph of

$$y = \frac{x^2 + x - 6}{x^2 - 1}, \qquad x \neq \pm 1$$

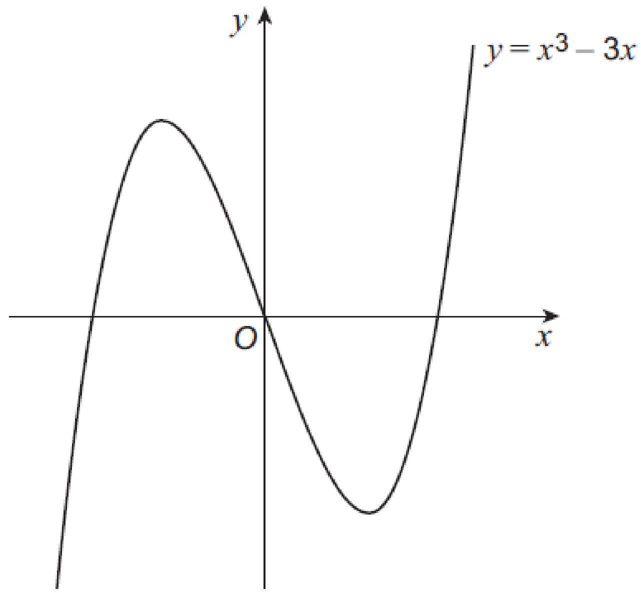
giving your answer in exact form.

No credit will be given for solutions based on differentiation.

[6 marks]

8. June/2019/Paper_1/No.14

The graph of $y = x^3 - 3x$ is shown below.



The two stationary points have x-coordinates of -1 and 1

The cubic equation

$$x^3 - 3x + p = 0$$

where p is a real constant, has the roots α, β and γ.

The roots α and β are **not** real.

(a) Explain why $\alpha + \beta = -\gamma$

[1 mark]

(b) Find the set of possible values for the real constant p.

[2 marks]

(c) $f(x) = 0$ is a cubic equation with roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$

(c) (i) Show that the constant term of $f(x)$ is $p + 2$

[3 marks]

(c) (ii) Write down the x-coordinates of the stationary points of $y = f(x)$

[1 mark]