AQA – Matrices – A2 Further Mathematics P2

1. June/2020/Paper_2/No.4

The matrices **A** and **B** are defined as follows:

$$\mathbf{A} = \begin{bmatrix} x+1 & 2 \\ x+2 & -3 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} x-4 & x-2 \\ 0 & -2 \end{bmatrix}$$

Show that there is a value of x for which $\mathbf{AB} = k\mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix and k is an integer to be found.

[3 marks]

2. June/2020/Paper_2/No.8

(a) Factorise $\begin{vmatrix} 2a+b+x & x+b & x^2+b^2 \\ 0 & a & -a^2 \\ a+b & b & b^2 \end{vmatrix}$ as fully as possible.

[6 marks]

(b) The matrix **M** is defined by

$$\mathbf{M} = \begin{bmatrix} 13+x & x+3 & x^2+9 \\ 0 & 5 & -25 \\ 8 & 3 & 9 \end{bmatrix}$$

Under the transformation represented by **M**, a solid of volume $0.625\,\text{m}^3$ becomes a solid of volume $300\,\text{m}^3$

Use your answer to part **(a)** to find the possible values of x.

[3 marks]

3. June/2020/Paper_2/No.9

The matrix $\mathbf{C} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, where a and b are positive real numbers,

and $\mathbf{C}^2 = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Use $\mathbf{C}$ to show that $\cos\frac{\pi}{12}$ can be written in the form $\frac{\sqrt{\sqrt{m}+n}}{2}$, where m and n are integers.

[7 marks]

4. June/2019/Paper_2/No.9

(a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$\mathbf{M} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{-3}{5} & \frac{13}{10} \end{bmatrix}$$

[5 marks]

(b) Find matrices **U** and **D** such that **D** is a diagonal matrix and $\mathbf{M} = \mathbf{UDU}^{-1}$

[2 marks]

(c) Given that $\mathbf{M}^n \to \mathbf{L}$ as $n \to \infty$, find the matrix **L**.

[4 marks]

(d) The transformation represented by **L** maps all points onto a line.

Find the equation of this line.

[2 marks]