# AQA – Integration – A2 Mathematics P1

**1.** June/2020/Paper_1/No.6

Four students, Tom, Josh, Floella and Georgia are attempting to complete the indefinite integral

$$\int \frac{1}{x}\,\mathrm{d}x \qquad \text{for } x > 0$$

Each of the students' solutions is shown below:

Tom $\int \frac{1}{x}\,\mathrm{d}x = \ln x$

Josh $\int \frac{1}{x}\,\mathrm{d}x = k\ln x$

Floella $\int \frac{1}{x}\,\mathrm{d}x = \ln Ax$

Georgia $\int \frac{1}{x}\,\mathrm{d}x = \ln x + c$

**(a) (i)** Explain what is wrong with Tom's answer.

**[1 mark]**

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**(a) (ii)** Explain what is wrong with Josh's answer.

**[1 mark]**

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**(b)** Explain why Floella and Georgia's answers are equivalent.

**[2 marks]**

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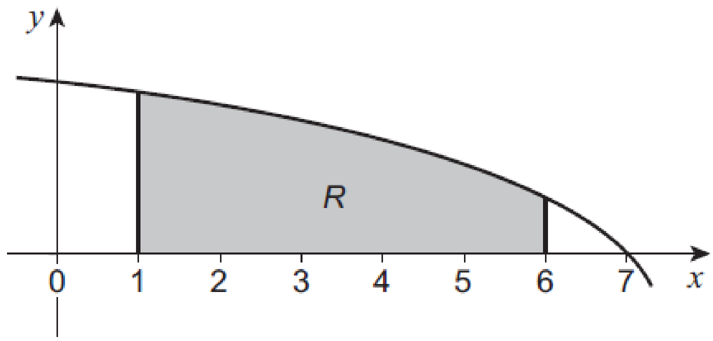
**2.** June/2020/Paper_1/No.11(a)

The region $R$ enclosed by the lines $x = 1$, $x = 6$, $y = 0$ and the curve

$$y = \ln(8 - x)$$

is shown shaded in **Figure 3** below.

**Figure 3**



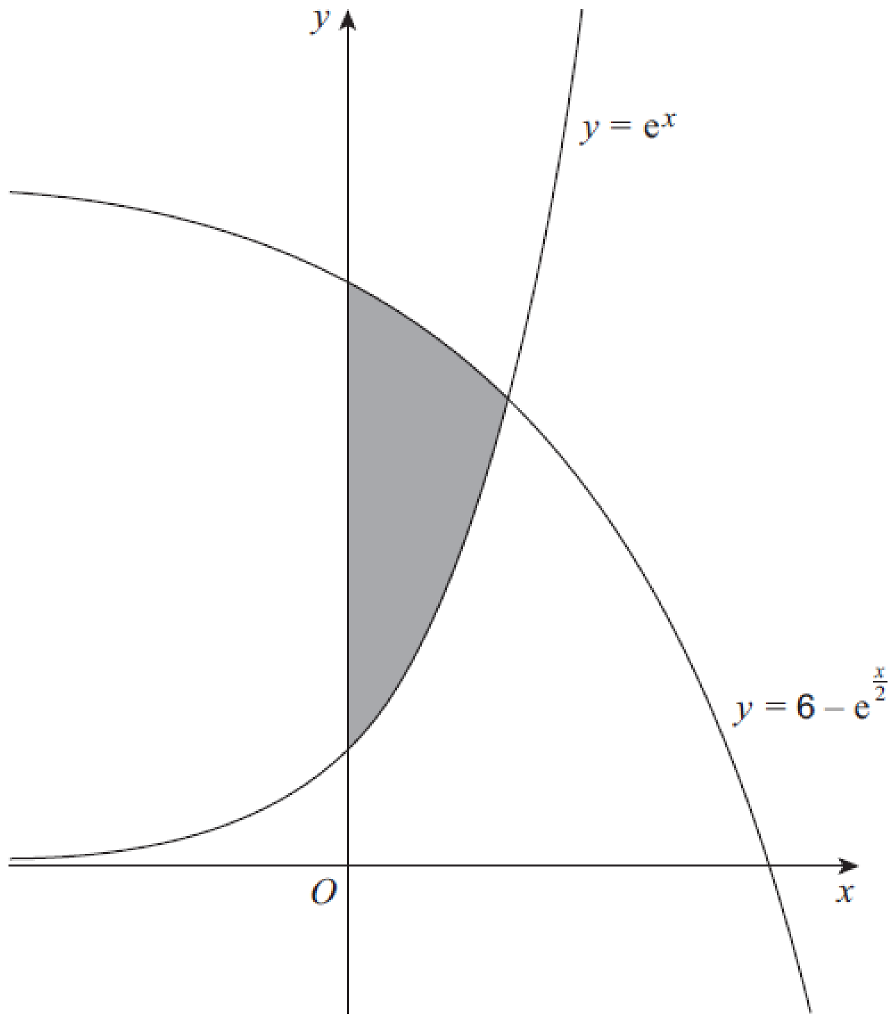

All distances are measured in centimetres.

**(a)** Use a single trapezium to find an approximate value of the area of the shaded region, giving your answer in $cm^2$ to two decimal places.

**[2 marks]**

**3.** June/2020/Paper_1/No.15

The region enclosed between the curves $y = e^x$, $y = 6 - e^{\frac{x}{2}}$ and the line $x = 0$ is shown shaded in the diagram below.




Show that the exact area of the shaded region is

$$6 \ln 4 - 5$$

Fully justify your answer.

**[10 marks]**

**4.** June/2020/Paper_1/No.9

Chloe is attempting to write $\dfrac{2x^2+x}{(x+1)(x+2)^2}$ as partial fractions, with constant numerators.

Her incorrect attempt is shown below.

Step 1 $\quad \dfrac{2x^2+x}{(x+1)(x+2)^2} \equiv \dfrac{A}{x+1} + \dfrac{B}{(x+2)^2}$

Step 2 $\quad 2x^2+x \equiv A(x+2)^2 + B(x+1)$

Step 3 $\quad$ Let $x=-1 \Rightarrow A=1$
Let $x=-2 \Rightarrow B=-6$

Answer $\quad \dfrac{2x^2+x}{(x+1)(x+2)^2} \equiv \dfrac{1}{x+1} - \dfrac{6}{(x+2)^2}$

**(a) (i)** By using a counter example, show that the answer obtained by Chloe cannot be correct.

**[2 marks]**

**(a) (ii)** Explain her mistake in Step 1.

**[1 mark]**

**(b)** Write $\dfrac{2x^2 + x}{(x+1)(x+2)^2}$ as partial fractions, with constant numerators.

**[4 marks]**

**5.** June/2019/Paper_1/No.16(b_c)

**(b)** Hence, show that

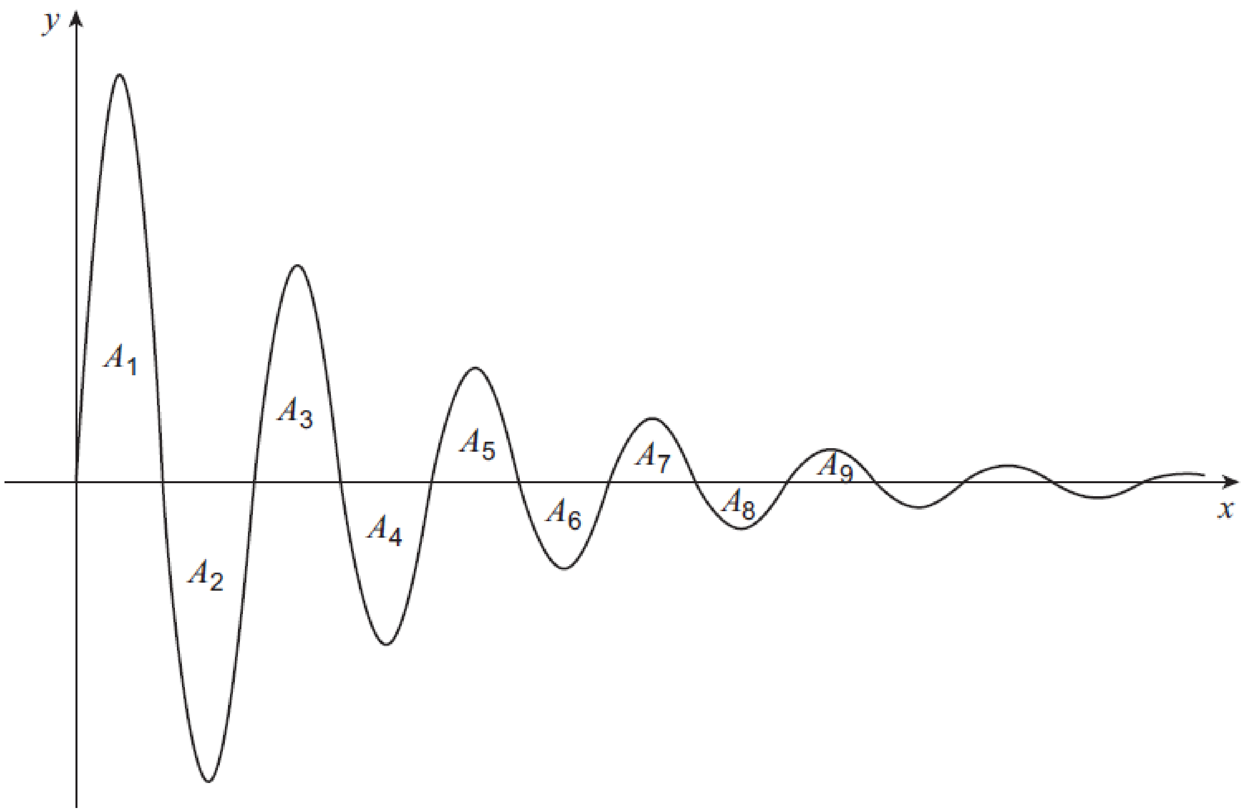
$$\int e^{-x} \sin x \, dx = ae^{-x}(\sin x + \cos x) + c$$

where $a$ is a rational number.

**[2 marks]**

**(c)** A sketch of the graph of $y = \mathrm{e}^{-x} \sin x$ for $x \geq 0$ is shown below.

The areas of the finite regions bounded by the curve and the $x$-axis are denoted by $A_1, A_2, \ldots, A_n, \ldots$




**(c) (i)** Find the exact value of the area $A_1$

**[3 marks]**

**(c) (ii)** Show that

$$\frac{A_2}{A_1} = e^{-\pi}$$

**[4 marks]**

**(c) (iii)** Given that

$$\frac{A_{n+1}}{A_n} = e^{-\pi}$$

show that the exact value of the total area enclosed between the curve and the $x$-axis is

$$\frac{1 + e^{\pi}}{2(e^{\pi} - 1)}$$

**[4 marks]**