

**AQA – Hyperbolic functions – AS Further Mathematics P1**

1. June/2020/Paper\_1/No.8

(a) Prove that

$$\tanh^{-1} x = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right)$$

**[5 marks]**

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(b) Prove that the graphs of

$$y = \sinh x \quad \text{and} \quad y = \cosh x$$

do not intersect.

[3 marks]

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## 2. June/2019/Paper\_1/No.6

Given that  $z_1 = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$  and  $z_2 = 2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

- (a) Find the value of  $|z_1 z_2|$  [1 mark]

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- (b) Find the value of  $\arg\left(\frac{z_1}{z_2}\right)$  [1 mark]

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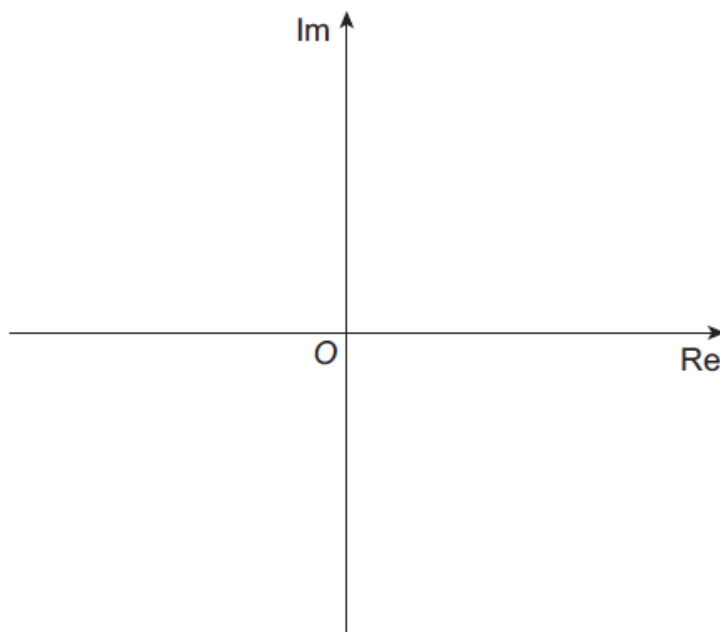


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- (c) Sketch  $z_1$  and  $z_2$  on the Argand diagram below, labelling the points as  $P$  and  $Q$  respectively. [2 marks]



- (d) A third complex number  $w$  satisfies both  $|w| = 2$  and  $-\pi < \arg w < 0$

Given that  $w$  is represented on the Argand diagram as the point  $R$ , find the angle  $\widehat{PRQ}$ .

Fully justify your answer.

[3 marks]

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## 3. June/2019/Paper\_1/No.9

(a) Saul is solving the equation

$$2 \cosh x + \sinh^2 x = 1$$

He writes his steps as follows:

$$2 \cosh x + \sinh^2 x = 1$$

$$2 \cosh x + 1 - \cosh^2 x = 1$$

$$2 \cosh x - \cosh^2 x = 0$$

$$\cosh x \neq 0 \quad \therefore 2 - \cosh x = 0$$

$$\cosh x = 2$$

$$x = \pm \cosh^{-1}(2)$$

Identify and explain the error in Saul's method.

[2 marks]

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(b) Anna is solving the differential equation

$$\sinh^2(2x) - 2 \cosh(2x) = 1$$

and finds the correct answers in the form  $x = \frac{1}{p} \cosh^{-1}(q + \sqrt{r})$ , where  $p$ ,  $q$  and  $r$  are integers.

Find the possible values of  $p$ ,  $q$  and  $r$ .

Fully justify your answer.

[5 marks]

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