AQA – Hyperbolic functions – A2 Further Mathematics P1

1. June/2020/Paper_1/No.12

   (a)   Use the definition of the cosh function to prove that

$$\cosh^{-1}\left(\frac{x}{a}\right) = \ln\left(\frac{x + \sqrt{x^2 - a^2}}{a}\right) \qquad \text{for } a > 0$$

[6 marks]

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(b) The formulae booklet gives the integral of $\dfrac{1}{\sqrt{x^2 - a^2}}$ as

$$\cosh^{-1}\left(\frac{x}{a}\right) \quad \text{or} \quad \ln\left(x + \sqrt{x^2 - a^2}\right) + c$$

Ronald says that this contradicts the result given in part **(a)**.

Explain why Ronald is wrong.

[2 marks]

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2. June/2020/Paper_1/No.14

(a)   Given that

$$\sinh(A + B) = \sinh A \cosh B + \cosh A \sinh B$$

express $\sinh(m + 1)x$ and $\sinh(m - 1)x$ in terms of $\sinh mx$, $\cosh mx$, $\sinh x$ and $\cosh x$

[1 mark]

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(b)   Hence find the sum of the series

$$C_n = \cosh x + \cosh 2x + \cdots + \cosh nx$$

in terms of $\sinh x$, $\sinh nx$ and $\sinh(n + 1)x$

[5 marks]

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3. June/2019/Paper_1/No.6

(a) Show that

$$\cosh^3 x + \sinh^3 x = \frac{1}{4}e^{mx} + \frac{3}{4}e^{nx}$$

where m and n are integers.

[3 marks]

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(b) Hence find $\cosh^6 x - \sinh^6 x$ in the form

$$\frac{a\cosh(kx) + b}{8}$$

where a, b and k are integers.

[5 marks]

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