AQA – Further vectors – A2 Further Mathematics P1

1. June/2020/Paper_1/No.7

Three planes have equations

$$\begin{aligned}(4k+1)x - \quad 3y + (k-5)z &= 3\\ (k-1)x + (3-k)y + \quad 2z &= 1\\ 7x - \quad 3y + \quad 4z &= 2\end{aligned}$$

(a) The planes do **not** meet at a unique point.

Show that $k = 4.5$ is one possible value of k, and find the other possible value of k.

[3 marks]

(b) For each value of k found in part **(a)**, identify the configuration of the given planes.

In each case fully justify your answer, stating whether or not the equations of the planes form a consistent system.

[4 marks]

2. June/2020/Paper_1/No.11

The lines l_1, l_2 and l_3 are defined as follows.

$$l_1 : \left(\mathbf{r} - \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix} \right) \times \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = \mathbf{0}$$

$$l_2 : \left(\mathbf{r} - \begin{bmatrix} -3 \\ 2 \\ 7 \end{bmatrix} \right) \times \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \mathbf{0}$$

$$l_3 : \left(\mathbf{r} - \begin{bmatrix} -5 \\ 12 \\ -4 \end{bmatrix} \right) \times \begin{bmatrix} 4 \\ 0 \\ 9 \end{bmatrix} = \mathbf{0}$$

(a) (i) Explain how you know that two of the lines are parallel.

[1 mark]

(a) (ii) Show that the perpendicular distance between these two parallel lines is 7.95 units, correct to three significant figures.

[5 marks]

(b) Show that the lines l_1 and l_3 meet, and find the coordinates of their point of intersection.

[5 marks]

3. June/2019/Paper_1/No.5

A plane has equation $\mathbf{r}.\begin{bmatrix}1\\1\\1\end{bmatrix} = 7$

A line has equation $\mathbf{r} = \begin{bmatrix}2\\0\\1\end{bmatrix} + \mu\begin{bmatrix}1\\0\\1\end{bmatrix}$

Calculate the acute angle between the line and the plane.

Give your answer to the nearest 0.1°

[3 marks]

4. June/2019/Paper_1/No.10

The points $A(5, -4, 6)$ and $B(6, -6, 8)$ lie on the line L. The point C is $(15, -5, 9)$.

(a) D is the point on L that is closest to C.

Find the coordinates of D.

[6 marks]

(b) Hence find, in exact form, the shortest distance from C to L.

[2 marks]

5. June/2019/Paper_1/No.12

Three planes have equations

$$4x - 5y + z = 8$$
$$3x + 2y - kz = 6$$
$$(k - 2)x + ky - 8z = 6$$

where k is a real constant.

The planes do **not** meet at a unique point.

(a) Find the possible values of k.

[3 marks]

(b) For each value of k found in part **(a)**, identify the configuration of the given planes.

Fully justify your answer, stating in each case whether or not the equations of the planes form a consistent system.

[5 marks]