AQA – Further algebra and functions – A2 Further Mathematics P1

1. June/2020/Paper_1/No.3

The quadratic equation $ax^2 + bx + c = 0$ $(a, b, c \in \mathbb{R})$ has real roots α and β.

One of the four statements below is incorrect.

Which statement is **incorrect**?

Tick (✓) **one** box.

[1 mark]

$c = 0 \Rightarrow \alpha = 0$ or $\beta = 0$ ☐

$c = a \Rightarrow \alpha$ is the reciprocal of β ☐

$b < 0$ and $c < 0 \Rightarrow \alpha > 0$ and $\beta > 0$ ☐

$b = 0 \Rightarrow \alpha = -\beta$ ☐

2. June/2020/Paper_1/No.5

H_1 is the locus of points such that the distance from the point (5, 0) is twice the distance from the line $x = 2$

(a) Show that the equation of H_1 can be written in the form

$$(x - 1)^2 - \frac{y^2}{q} = r$$

where q and r are integers.

[5 marks]

(b) H_2 is the hyperbola

$$x^2 - y^2 = 4$$

Describe fully a sequence of two transformations which maps the graph of H_2 onto the graph of H_1

[4 marks]

3. June/2020/Paper_1/No.8

The three roots of the equation

$$4x^3 - 12x^2 - 13x + k = 0$$

where k is a constant, form an arithmetic sequence.

Find the roots of the equation.

[6 marks]

4. June/2020/Paper_1/No.9

The function f is defined by

$$f(x) = \frac{x(x+3)}{x+4} \qquad (x \in \mathbb{R}, x \neq -4)$$

(a) Find the interval (a, b) in which $f(x)$ does not take any values.

Fully justify your answer.

[5 marks]

(b) Find the coordinates of the two stationary points of the graph of $y = \mathrm{f}(x)$

[2 marks]

(c) Show that the graph of $y = \mathrm{f}(x)$ has an oblique asymptote and find its equation.

[2 marks]

5. June/2019/Paper_1/No.1

Which one of these functions has the set $\{x : |x| < 1\}$ as its greatest possible domain?

Circle your answer.

[1 mark]

$\cosh x$ $\cosh^{-1} x$ $\tanh x$ $\tanh^{-1} x$

6. June/2019/Paper_1/No.2

The first two non-zero terms of the Maclaurin series expansion of $f(x)$ are x and $-\frac{1}{2}x^3$

Which one of the following could be $f(x)$?

Circle your answer.

[1 mark]

$xe^{\frac{1}{2}x^2}$ $\frac{1}{2}\sin 2x$ $x\cos x$ $(1+x^3)^{-\frac{1}{2}}$

7. June/2019/Paper_1/No.13

The equation $z^3 + kz^2 + 9 = 0$ has roots α, β and γ.

(a) (i) Show that

$$\alpha^2 + \beta^2 + \gamma^2 = k^2$$

[3 marks]

(a) (ii) Show that

$$\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 = -18k$$

[4 marks]

(b) The equation $9z^3 - 40z^2 + rz + s = 0$ has roots $\alpha\beta + \gamma$, $\beta\gamma + \alpha$ and $\gamma\alpha + \beta$.

(b) (i) Show that

$$k = -\frac{40}{9}$$

[1 mark]

(b) (ii) Without calculating the values of α, β and γ, find the value of s.

Show working to justify your answer.

[6 marks]