AQA – Differentiation – A2 Mathematics P1

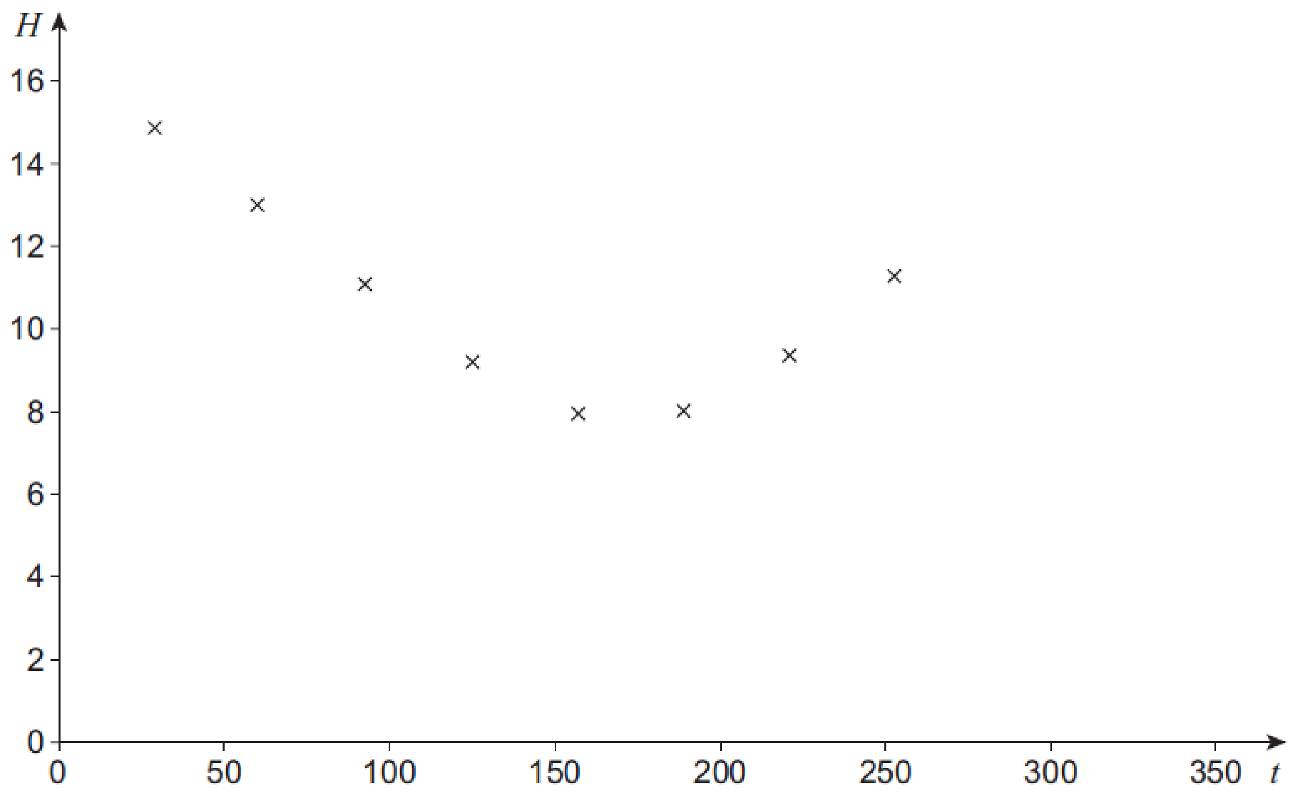
1. June/2020/Paper_1/No.8

Mike, an amateur astronomer who lives in the South of England, wants to know how the number of hours of darkness changes through the year.

On various days between February and September he records the length of time, H hours, of darkness along with t, the number of days after 1 January.

His results are shown in **Figure 1** below.

Figure 1



Mike models this data using the equation

$$H = 3.87\sin\left(\frac{2\pi(t + 101.75)}{365}\right) + 11.7$$

(a) Find the minimum number of hours of darkness predicted by Mike's model. Give your answer to the nearest minute.

[2 marks]

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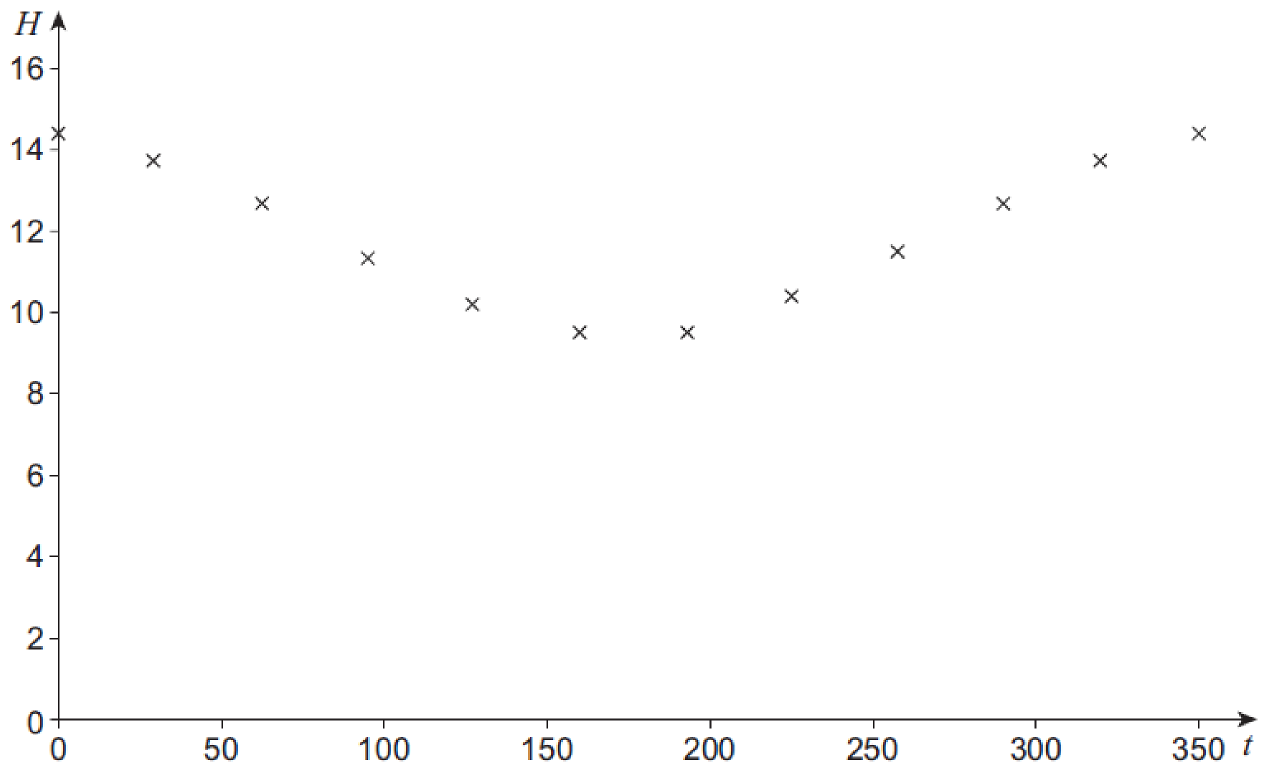
(b) Find the maximum number of consecutive days where the number of hours of darkness predicted by Mike's model exceeds 14

[3 marks]

(c) Mike's friend Sofia, who lives in Spain, also records the number of hours of darkness on various days throughout the year.

Her results are shown in **Figure 2** below.

Figure 2



Sofia attempts to model her data by refining Mike's model.

She decides to increase the 3.87 value, leaving everything else unchanged.

Explain whether Sofia's refinement is appropriate.

[2 marks]

2. June/2020/Paper_1/No.12

A curve C has equation

$$x^3 \sin y + \cos y = Ax$$

where A is a constant.

C passes through the point $P\left(\sqrt{3}, \frac{\pi}{6}\right)$

(a) Show that $A = 2$

[2 marks]

(b) (i) Show that $\frac{dy}{dx} = \frac{2 - 3x^2 \sin y}{x^3 \cos y - \sin y}$

[5 marks]

(b) (ii) Hence, find the gradient of the curve at P.

[2 marks]

(b) (iii) The tangent to C at P intersects the x-axis at Q.

Find the exact x-coordinate of Q.

[4 marks]

3. June/2019/Paper_1/No.2

Given $y = e^{kx}$, where k is a constant, find $\dfrac{dy}{dx}$

Circle your answer.

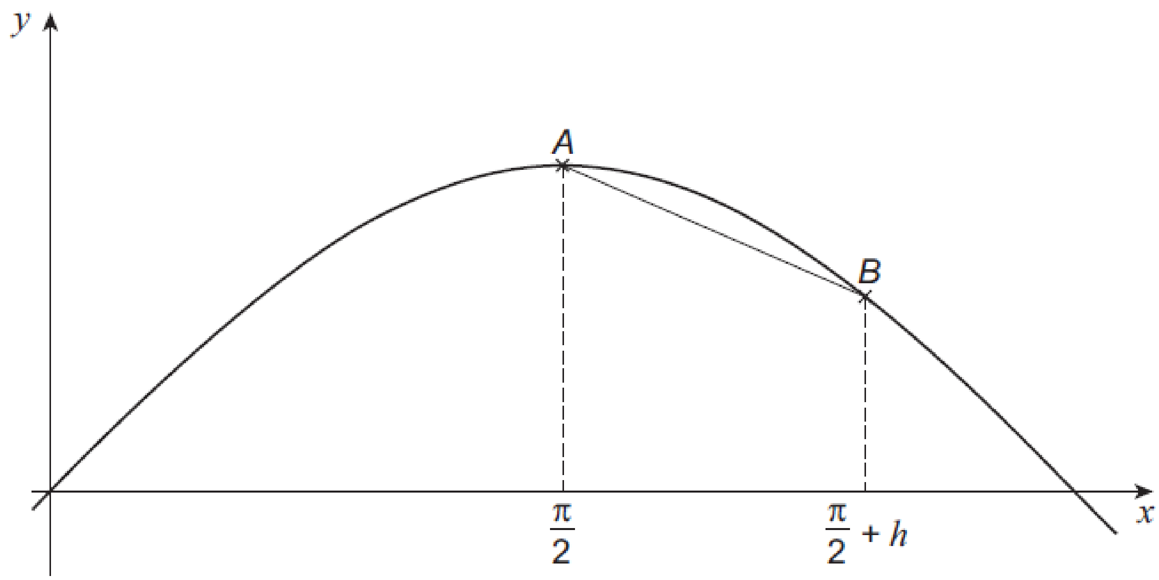
[1 mark]

$\dfrac{dy}{dx} = e^{kx}$ $\quad\quad$ $\dfrac{dy}{dx} = ke^{kx}$ $\quad\quad$ $\dfrac{dy}{dx} = kxe^{kx-1}$ $\quad\quad$ $\dfrac{dy}{dx} = \dfrac{e^{kx}}{k}$

4. June/2019/Paper_1/No.11

Jodie is attempting to use differentiation from first principles to prove that the gradient of $y = \sin x$ is zero when $x = \dfrac{\pi}{2}$

Jodie's teacher tells her that she has made mistakes starting in Step 4 of her working. Her working is shown below.



Step 1 Gradient of chord $AB = \dfrac{\sin\left(\frac{\pi}{2}+h\right) - \sin\left(\frac{\pi}{2}\right)}{h}$

Step 2 $= \dfrac{\sin\left(\frac{\pi}{2}\right)\cos(h) + \cos\left(\frac{\pi}{2}\right)\sin(h) - \sin\left(\frac{\pi}{2}\right)}{h}$

Step 3 $= \sin\left(\frac{\pi}{2}\right)\left(\dfrac{\cos(h) - 1}{h}\right) + \cos\left(\frac{\pi}{2}\right)\dfrac{\sin(h)}{h}$

Step 4 For gradient of curve at A,

let $h = 0$ then

$\dfrac{\cos(h) - 1}{h} = 0$ and $\dfrac{\sin(h)}{h} = 0$

Step 5 Hence the gradient of the curve at A is given by

$\sin\left(\frac{\pi}{2}\right) \times 0 + \cos\left(\frac{\pi}{2}\right) \times 0 = 0$

Complete Steps 4 and 5 of Jodie's working below, to correct her proof.

[4 marks]

Step 4 For gradient of curve at A,

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Step 5 Hence the gradient of the curve at A is given by

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5. June/2019/Paper_1/No.13

A curve, C, has equation

$$y = \frac{e^{3x-5}}{x^2}$$

Show that C has exactly one stationary point.

Fully justify your answer.

[7 marks]

6. June/2019/Paper_1/No.15

(a) At time t hours **after a high tide**, the height, h metres, of the tide and the velocity, v knots, of the tidal flow can be modelled using the parametric equations

$$v = 4 - \left(\frac{2t}{3} - 2\right)^2$$

$$h = 3 - 2\sqrt[3]{t - 3}$$

High tides and low tides occur alternately when the velocity of the tidal flow is zero.

A high tide occurs at 2 am.

(a) (i) Use the model to find the height of this high tide.

[1 mark]

(a) (ii) Find the time of the first **low** tide after 2 am.

[3 marks]

(a) (iii) Find the height of this low tide.

[1 mark]

(b) Use the model to find the height of the tide when it is flowing with maximum velocity.

[3 marks]

(c) Comment on the validity of the model.

[2 marks]

7. June/2019/Paper_1/No.16(a)

(a) $y = e^{-x}(\sin x + \cos x)$

Find $\dfrac{dy}{dx}$

Simplify your answer.

[3 marks]

8. June/2019/Paper_1/No.10

The volume of a spherical bubble is increasing at a constant rate.

Show that the rate of increase of the radius, r, of the bubble is inversely proportional to r^2

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

[4 marks]