AQA – Differential equations – A2 Further Mathematics P2

1. June/2020/Paper_2/No.12(b)

(b) A small object is initially at rest. The subsequent motion of the object is modelled by the differential equation

$$\frac{dv}{dt} + v = 5e^t \sin t$$

where v is the velocity at time t.

Find the speed of the object when $t = 2\pi$, giving your answer in exact form.

[6 marks]

2. June/2020/Paper_2/No.13

Charlotte is trying to solve this mathematical problem:

> Find the general solution of the differential equation
>
> $$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 10e^{-2x}$$

Charlotte's solution starts as follows:

> Particular integral: $y = \lambda e^{-2x}$
>
> so
>
> $$\frac{dy}{dx} = -2\lambda e^{-2x}$$
>
> and
>
> $$\frac{d^2y}{dx^2} = 4\lambda e^{-2x}$$

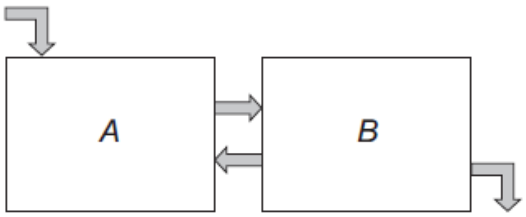
(a) Show that Charlotte's method will fail to find a particular integral for the differential equation.

[2 marks]

(b) Explain how Charlotte should have started her solution differently and find the general solution of the differential equation.

[8 marks]

3. June/2019/Paper_2/No.15



Two tanks, *A* and *B*, each have a capacity of 800 litres.

At time $t = 0$ both tanks are full of pure water.

When $t > 0$, water flows in the following ways:

- Water with a salt concentration of μ grams per litre flows into tank *A* at a constant rate
- Water flows from tank *A* to tank *B* at a rate of 16 litres per minute
- Water flows from tank *B* to tank *A* at a rate of r litres per minute
- Water flows out of tank *B* through a waste pipe
- The amount of water in each tank remains at 800 litres.

At time t minutes $(t \geq 0)$ there are x grams of salt in tank *A* and y grams of salt in tank *B*.

This system is represented by the coupled differential equations

$$\frac{dx}{dt} = 36 - 0.02x + 0.005y \qquad (1)$$

$$\frac{dy}{dt} = 0.02x - 0.02y \qquad (2)$$

(a) Find the value of r.

[2 marks]

(b) Show that $\mu = 3$

[3 marks]

(c) Solve the coupled differential equations to find both x and y in terms of t.

[9 marks]