AQA – Differential equations – A2 Further Mathematics P1

1. June/2020/Paper_1/No.10

(a) Find the general solution of the differential equation

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{x+3}{x(x-1)(x^2+3)} \qquad (x > 1)$$

[8 marks]

(b) Find the particular solution for which $y = 0$ when $x = 3$

Give your answer in the form $y = \mathrm{f}(x)$

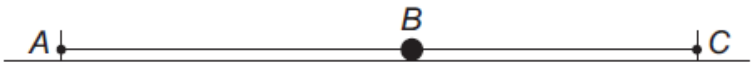
[2 marks]

2. June/2020/Paper_1/No.13

Two light elastic strings each have one end attached to a particle B of mass $3c$ kg, which rests on a smooth horizontal table.

The other ends of the strings are attached to the fixed points A and C, which are 8 metres apart.

ABC is a horizontal line.



String AB has a natural length of 4 metres and a stiffness of $5c$ newtons per metre.

String BC has a natural length of 1 metre and a stiffness of c newtons per metre.

The particle is pulled a distance of $\frac{1}{3}$ metre from its equilibrium position towards A, and released from rest.

(a) Show that the particle moves with simple harmonic motion.

[8 marks]

(b) Find the speed of the particle when it is at a point P, a distance $\frac{1}{4}$ metre from the equilibrium position. Give your answer to two significant figures.

[4 marks]

3. June/2019/Paper_1/No.11

Find the general solution of the differential equation

$$x\frac{dy}{dx} - 2y = \frac{x^3}{\sqrt{4 - 2x - x^2}}$$

where $0 < x < \sqrt{5} - 1$

[7 marks]

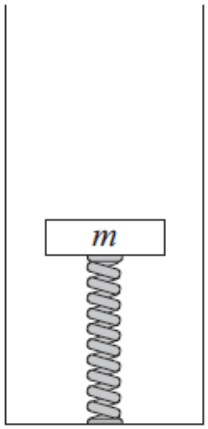
4. June/2019/Paper_1/No.14

In this question use $g = 10\,\text{m s}^{-2}$

A light spring is attached to the base of a long tube and has a mass m attached to the other end, as shown in the diagram.

The tube is filled with oil.

When the compression of the spring is ε metres, the thrust in the spring is $9m\varepsilon$ newtons.



The mass is held at rest in a position where the compression of the spring is $\frac{20}{9}$ metres.

The mass is then released from rest. During the subsequent motion the oil causes a resistive force of $6mv$ newtons to act on the mass, where $v\,\text{m s}^{-1}$ is the speed of the mass.

At time t seconds after the mass is released, the displacement of the mass above its starting position is x metres.

(a) Find x in terms of t.

[10 marks]

(b) State, giving a reason, the type of damping which occurs.

[1 mark]