

**AQA – Complex numbers – A2 Further Mathematics P1****1. June/2020/Paper\_1/No.4**

It is given that  $1 - 3i$  is one root of the quartic equation

$$z^4 - 2z^3 + pz^2 + rz + 80 = 0$$

where  $p$  and  $r$  are real numbers.

- (a) Express  $z^4 - 2z^3 + pz^2 + rz + 80$  as the product of two quadratic factors with real coefficients.

**[4 marks]**

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(b) Find the value of  $p$  and the value of  $r$ .

[2 marks]

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2. June/2020/Paper\_1/No.6(a\_c)

Let  $w$  be the root of the equation  $z^7 = 1$  that has the smallest argument  $\alpha$  in the interval  $0 < \alpha < \pi$

(a) Prove that  $w^n$  is also a root of the equation  $z^7 = 1$  for any integer  $n$ .

[1 mark]

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(b) Prove that  $1 + w + w^2 + w^3 + w^4 + w^5 + w^6 = 0$

[2 marks]

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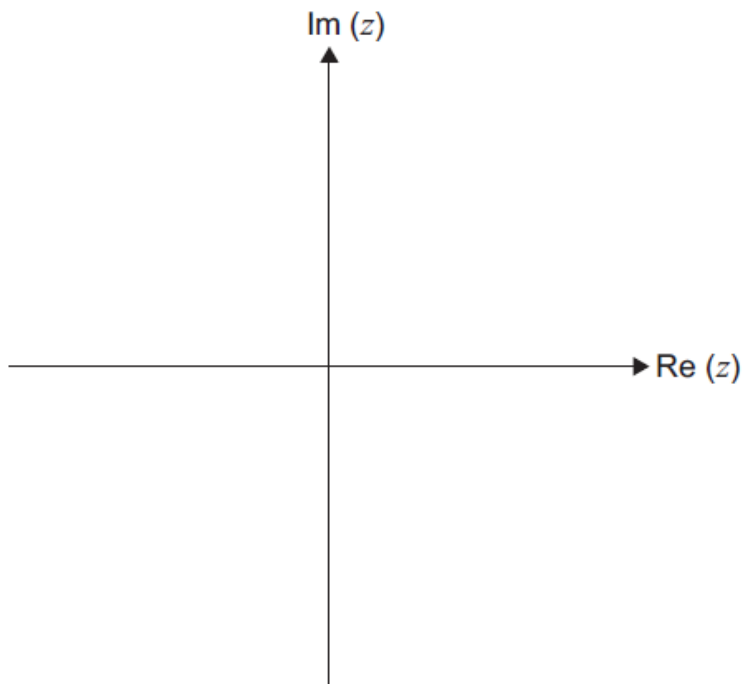
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- (c) Show the positions of  $w$ ,  $w^2$ ,  $w^3$ ,  $w^4$ ,  $w^5$ , and  $w^6$  on the Argand diagram below. [2 marks]



3. June/2019/Paper\_1/No.4

Solve the equation  $2z - 5iz^* = 12$

[4 marks]

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## 4. June/2019/Paper\_1/No.8

(a) If  $z = \cos \theta + i \sin \theta$ , use de Moivre's theorem to prove that

$$z^n - \frac{1}{z^n} = 2i \sin n\theta$$

[3 marks]

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(b) Express  $\sin^5 \theta$  in terms of  $\sin 5\theta$ ,  $\sin 3\theta$  and  $\sin \theta$

[4 marks]

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(c) Hence show that

$$\int_0^{\frac{\pi}{3}} \sin^5 \theta \, d\theta = \frac{53}{480}$$

[3 marks]

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5. June/2019/Paper\_1/No.9

- (a) Solve the equation  $z^3 = \sqrt{2} - \sqrt{6}i$ , giving your answers in the form  $re^{i\theta}$  where  $r > 0$  and  $0 \leq \theta < 2\pi$

[5 marks]

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- (b) The transformation represented by the matrix  $\mathbf{M} = \begin{bmatrix} 5 & 1 \\ 1 & 3 \end{bmatrix}$  acts on the points on an Argand Diagram which represent the roots of the equation in part (a).

Find the exact area of the shape formed by joining the transformed points.

[4 marks]

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