AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________
I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 2 Mechanics

Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

J U N 2 1 7 3 6 6 2 M 0 1

PB/Jun21/E4

7366/2M

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Answer **all** questions in the spaces provided.

1 A light spring of natural length 0.6 metres is compressed to a length of 0.4 metres by a force of 20 newtons.

The stiffness of the spring is $k\,\mathrm{N\,m^{-1}}$

Find k

Circle your answer.

[1 mark]

20 50 100 200

$F = -k\Delta x$
$20 = -k(0.4 - 0.6)$
$20 = -k(-0.2)$
$20 = 0.2k$
$\Rightarrow k = \frac{20}{0.2}$
$= 100$

2 State the dimensions of force.

Circle your answer.

[1 mark]

MLT ML^2T MLT^{-1} MLT^{-2}

$F = ma$

dimensions for $m = M$
$a = LT^{-2}$

$\therefore ma = MLT^{-2}$

$\therefore F = MLT^{-2}$

Do not write outside the box

3 **Use g as $9.8\,\mathrm{m\,s^{-2}}$ in this question.**

A pump is used to pump water out of a pool.

The pump raises the water through a vertical distance of 5 metres and then ejects it through a pipe.

The pump works at a constant rate of 400 W

Over a period of 50 seconds, 300 litres of water are pumped out of the pool and the water is ejected with speed $v\,\mathrm{m\,s^{-1}}$

The mass of 1 litre of water is 1 kg

3 (a) Find the gain in the potential energy of the 300 litres of water.

[1 mark]

Gain in Potential energy = mgh

$= 300 \times 9.8 \times 5$

$= 14700$ J

3 (b) Calculate v

[4 marks]

Using the work-energy principle

Work done = gain in Kinetic Energy + gain in Potential Energy

Work done over a period of 50 sec = Power × time

$= 400 \times 50$

$= 20000$ J

gain ~~in~~ KE $= \frac{1}{2} \times m \times v^2$

$= \frac{1}{2} \times 300 \times v^2$

$\Rightarrow \quad 20000 = \frac{1}{2} \times 300 \times v^2 + 14700$

$\left(\frac{20000 - 14700}{150}\right) = \frac{150 v^2}{150}$

$v^2 = 35.3333$

$v = \sqrt{35.333}$

$= 5.944$

$\therefore v = 5.944\ \mathrm{ms^{-1}}$

Turn over ▶

Do not write outside the box

4 A cyclist in a road race is travelling around a bend on a horizontal circular path of radius 15 metres and is prevented from skidding by a frictional force.

The frictional force has a maximum value of 500 newtons.

The total mass of the cyclist and his cycle is 75 kg

Assume that the cyclist travels at a constant speed.

4 (a) Work out the greatest speed, in km h^{-1}, at which the cyclist can travel around the bend.

[4 marks]

Radial force $= \frac{mv^2}{r}$

$500 = \frac{75 \times v^2}{15}$

$(500 \times 15) = 75v^2$

$\frac{7500}{75} = \frac{\cancel{75}v^2}{\cancel{75}}$

$v^2 = 100 \Rightarrow v = \sqrt{100} = 10$

$\therefore \quad v = 10\ ms^{-1}$

Converting $10 ms^{-1}$ to $Kmhr^{-1}$

$\left(\frac{10 \times 3600}{1000}\right) Kmh^{-1}$

$= \left(\frac{36000}{1000}\right) = 36$

$\therefore$ greatest speed $= 36\ Kmh^{-1}$

4 (b) With reference to the surface of the road, describe one limitation of the model.

[1 mark]

The road surface may not be uniform.

Do not write outside the box

5 A ball is thrown vertically upwards with speed u so that at time t its displacement s is given by the formula

$$s = ut - \frac{gt^2}{2}$$

Use dimensional analysis to show that this formula is dimensionally consistent.

Fully justify your answer.

[4 marks]

Dimensional analysis is the study of the relation between physical quantities based on their units and dimensions. That is mass (M), Length (L) and Time (T)

dimensions of $s = L$

$u = LT^{-1}$

$t = T$

$g = LT^{-2}$

Recall $\left(\frac{1}{2}\right)$ is dimensionless

Therefore $ut = (LT^{-1})T = L$

$\frac{gt^2}{2} = (LT^{-2})T^2 = L$

$\Rightarrow (ut) = \left(\frac{gt^2}{2}\right) = s = L$

Therefore the formula is dimensionally consistent.

Turn over for the next question

Do not write outside the box

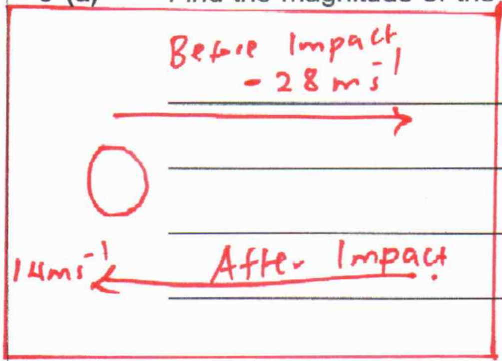
6 A ball of mass 0.15 kg is hit directly by a vertical cricket bat.

Immediately before the impact, the ball is travelling horizontally with speed $28\,\text{m s}^{-1}$

Immediately after the impact, the ball is travelling horizontally with speed $14\,\text{m s}^{-1}$ in the opposite direction.

6 (a) Find the magnitude of the impulse exerted by the bat on the ball.

[2 marks]



Impulse $I = mv - mu$

$I = (0.15 \times 14) - (0.15 \times -28)$

$= 2.1 - (-4.2)$

$= 2.1 + 4.2$

$= 6.3$

$\therefore$ Impulse $= 6.3\ Ns$

6 (b) In a simple model the force, F newtons, exerted by the bat on the ball, t seconds after the initial impact, is given by

$$F = 10kt\,(0.05 - t)$$

where k is a constant.

Given the ball is in contact with the bat for 0.05 seconds, find the value of k

[3 marks]

Impulse $= \int_0^{0.05} 10kt(0.05 - t)\,dt$

$6.3 = k\int_0^{0.05} 10t(0.05 - t)\,dt$

$6.3 = k\int_0^{0.05} 0.5t - 10t^2\,dt$

$6.3 = k\left[\frac{0.5t^2}{2} - \frac{10t^3}{3}\right]_0^{0.05}$

$6.3 = k\left[\frac{0.5(0.05^2)}{2} - \frac{10(0.05^3)}{3}\right]$

$6.3 = \frac{1}{4800}k$

$\Rightarrow K = 6. \times 4800$

$= 30240$

$\therefore K = 30240$

Turn over ▶

Do not write outside the box

7 **Use g as $9.81\,\mathrm{m\,s^{-2}}$ in this question.**

A light elastic string has one end attached to a fixed point A on a smooth plane inclined at 25° to the horizontal.

The other end of the string is attached to a wooden block of mass 2.5 kg, which rests on the plane.

The elastic string has natural length 3 metres and modulus of elasticity 125 newtons.

The block is pulled down the line of greatest slope of the plane to a point 4.5 metres from A and then released.

7 (a) Find the elastic potential energy of the string at the point when the block is released.

[1 mark]

Elastic Potential Energy (EPE) $= \dfrac{\lambda x^2}{2l}$

$x = (4.5 - 3) = 1.5$

EPE $= \dfrac{125 \times (1.5)^2}{2 \times 3} = \dfrac{281.25}{6}$

$= 46.875 \simeq 46.9$

$\therefore$ EPE $= 46.9\,\mathrm{J}$

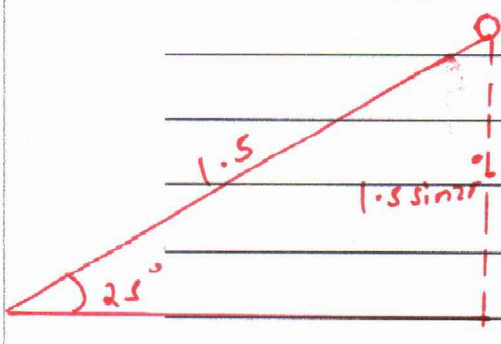
7 (b) Calculate the speed of the block when the string becomes slack.

[4 marks]

Let v = speed of the block when the string becomes slack.

Increase in height $= 1.5 \sin 25°$

By conservation of energy:



EPE lost = KE gained + PE gained

$46.875 = \frac{1}{2}mv^2 + mgh$

$46.875 = \left(\frac{1}{2} \times 2.5 \times v^2\right) + \left(2.5 \times 9.8 \times 1.5 \sin 25\right)$

$46.875 = 1.25v^2 + 15.531$

$\dfrac{1.25v^2}{1.25} = \dfrac{31.344}{1.25}$

$v^2 = 25.0752$

$v = \sqrt{25.0752}$

$= 5.0075 \simeq 5.01$

$\therefore v = 5.01\,\mathrm{m\,s^{-1}}$

Do not write outside the box

7 (c) Determine whether the block reaches the point *A* in the subsequent motion, commenting on any assumptions that you make.

[3 marks]

PE gained from the start to A = mgh

$h = 4.5 \sin 25°$

$= 2.5 \times 9.8 \times 4.5 \sin 25°$

$= 46.594$ J

EPE at start = 46.875 J

$\Rightarrow 46.594 < 46.875$

The model assumes that all energy is conserved.

The model assumes that there is no air resistance.

$\Rightarrow$ If there is no air resistance, the block will reach A as there is enough initial energy.

Turn over for the next question

Do not write outside the box

8 Two spheres A and B are free to move on a smooth horizontal surface.

The masses of A and B are 2 kg and 3 kg respectively.

Both A and B are initially at rest.

Sphere A is set in motion directly towards sphere B with speed $4\,\text{m s}^{-1}$ and subsequently collides with sphere B

The coefficient of restitution between the spheres is e

8 (a) (i) Show that the speed of B immediately after the collision is

$$\frac{8(1+e)}{5}$$

[4 marks]

Let velocity of A after collision $= V$

velocity of B after collision $= \omega$

By Conservation of Linear momentum

$(2\times4) + (3\times0) = (2\times V) + (3\times\omega)$

$8 = 2V + 3\omega \quad \ldots\ldots (i)$

Newton's Law of restitution:

$e = \dfrac{\text{Speed of separation}}{\text{speed of approach}}$

$e = \dfrac{V-\omega}{0-4}$

$-4e = V - \omega$

$\Rightarrow V = -4e + \omega \quad \ldots\ldots (ii)$

Replacing in (i)

$8 = 2(-4e + \omega) + 3\omega$

$8 = -8e + 2\omega + 5\omega$

$\dfrac{8 + 8e}{5} = \dfrac{\cancel{8}\omega}{\cancel{8}}$

$\Rightarrow \omega = \dfrac{8(1+e)}{5}$

$\therefore$ Speed of $B = \dfrac{8(1+e)}{5}$

8 (a) (ii) Find an expression, in terms of e, for the velocity of A immediately after the collision.

[2 marks]

$\omega = \dfrac{8(1+e)}{5}$

$V = -4e + \omega$

$= -4e + \dfrac{8(1+e)}{5}$

$= -4e + \dfrac{8}{5} + \dfrac{8}{5}e$

$= \dfrac{8}{5} - \dfrac{12}{5}e$

$= \dfrac{4(2-3e)}{5}$

$\therefore V = \dfrac{4(2-3e)}{5}$

Do not write outside the box

8 (b) It is given that the spheres both move in the **same** direction after the collision.

Find the range of possible values of e

[2 marks]

$\frac{4(2-3e)}{5} > 0$

$\frac{8}{5} - \frac{12e}{5} > 0$

$\Rightarrow \frac{8}{5} > \frac{12e}{5}$

$\frac{8}{5} \div \frac{12}{5}$, $\frac{12e}{5} \div \frac{12}{5}$

$\Rightarrow \frac{8}{12} > e$

$\frac{2}{3} > e$

$\Rightarrow e < 2/3$

$\therefore 0 \leq e < 2/3$

8 (c) (i) The impulse of sphere A on sphere B is I
The impulse of sphere B on sphere A is J

Given that the collision is perfectly inelastic, find the value of $I + J$

[1 mark]

The impulse of A and B is equal in magnitude but opposite in direction.

$\Rightarrow I = -J$

$\therefore I + J = 0$

8 (c) (ii) State, giving a reason for your answer, whether the value found in part **(c)(i)** would change if the collision was **not** perfectly inelastic.

[2 marks]

I + J will not change because Impulses are equal in magnitude and opposite in direction, therefore the impulse will always sum to zero.

END OF QUESTIONS