AQA

Please write clearly in block capitals.

Centre number						Candidate number				

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

I declare this is my own work.

A-level
MATHEMATICS
Paper 1

Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
TOTAL	

J U N 2 1 7 3 5 7 1 0 1

PB/Jun21/E5

7357/1

Answer **all** questions in the spaces provided.

1 State the set of values of x which satisfies the inequality

$$(x-3)(2x+7) > 0$$

Tick (✓) **one** box.

[1 mark]

$\left\{x : -\frac{7}{2} < x < 3\right\}$ ☐

$\left\{x : x < -3 \text{ or } x > \frac{7}{2}\right\}$ ☐

$\left\{x : x < -\frac{7}{2} \text{ or } x > 3\right\}$ ☑

$\left\{x : -3 < x < \frac{7}{2}\right\}$ ☐

$x > 3$

or

$\frac{2x}{2} < -\frac{7}{2}$

$x < -\frac{7}{2}$

2 Given that $y = \ln(5x)$

find $\frac{dy}{dx}$

Circle your answer.

[1 mark]

$\frac{dy}{dx} = \frac{1}{x}$ $\qquad$ $\frac{dy}{dx} = \frac{1}{5x}$ $\qquad$ $\frac{dy}{dx} = \frac{5}{x}$ $\qquad$ $\frac{dy}{dx} = \ln 5$

$y = \ln(5x)$

$\frac{dy}{dx} = \frac{1}{5x} \cdot 5$

$= \frac{5}{5x} = \frac{1}{x}$

$\therefore \frac{dy}{dx} = \frac{1}{x}$

Do not write outside the box

3 A geometric sequence has a sum to infinity of -3

A second sequence is formed by multiplying each term of the original sequence by -2

What is the sum to infinity of the new sequence?

Circle your answer.

[1 mark]

The sum to infinity does not exist -6 -3 6

4 Millie is attempting to use proof by contradiction to show that the result of multiplying an irrational number by a non-zero rational number is always an irrational number.

Select the assumption she should make to start her proof.

Tick (✓) **one** box.

[1 mark]

Every irrational multiplied by a non-zero rational is irrational. ☐

Every irrational multiplied by a non-zero rational is rational. ☐

There exists a non-zero rational and an irrational whose product is irrational. ☐

There exists a non-zero rational and an irrational whose product is rational. ✓

Turn over for the next question

Do not write outside the box

5 The line L has equation

$$3y - 4x = 21$$

The point P has coordinates (15, 2)

5 (a) Find the equation of the line perpendicular to L which passes through P.

[2 marks]

If two lines are perpendicular then their gradients $m_1 \times m_2 = 1$

$3y - 4x = 21$

$3y = 4x + 21$

$y = \frac{4}{3}x + 7$

Gradient $= \frac{4}{3}$

Gradient of $L_2 = \frac{4}{3} \times m_2 = -1$

$m_2 = -1 \times \frac{3}{4} = -\frac{3}{4}$

$\frac{y-2}{x-15} = -\frac{3}{4}$

$y - 2 = -\frac{3}{4}(x - 15)$

$4y - 8 = -3x + 45$

$4y + 3x = 53$

or

$y = -\frac{3}{4}x + \frac{53}{4}$

5 (b) Hence, find the shortest distance from P to L.

[4 marks]

$3y - 4x = 21$ ---- (i)

$4y + 3x = 53$ ---- (ii)

Solving (i) and (ii) simultaneously

from (i) $3y = 21 + 4x$

$y = \frac{21 + 4x}{3}$

$y = 7 + \frac{4}{3}x$

Replacing y in (ii)

$4\left(7 + \frac{4}{3}x\right) + 3x = 53$

$28 + \frac{16}{3}x + 3x = 53$

$\frac{25}{3}x = 25$

$\Rightarrow x = 25 \times \frac{3}{25} = 3$

$\therefore x = 3$

$y = 7 + \frac{4}{3}(3)$

$= 7 + 4 = 11$

$\therefore y = 11$

Distance from a point in L (3, 11) and P (15, 2)

Distance $= \sqrt{x_1 - x_2 + y_1 + y_2}$

$= \sqrt{(3 - 15)^2 + (11 - 2)^2}$

$= \sqrt{144 + 81}$

$= \sqrt{225} = 15$

$\therefore$ Distance $= 15$

Do not write outside the box

6 (a) The ninth term of an arithmetic series is 3

The sum of the first n terms of the series is S_n and $S_{21} = 42$

Find the first term and common difference of the series.

[4 marks]

n^{th} term $= a + (n-1)d$

9^{th} term $= a + (9-1)d = 3$

$a + 8d = 3$ ------ (i)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{21} = \frac{21}{2}[2a + (21-1)d] = 42$

$= \frac{21}{2}(2a + 20d) = 42$

$= 21a + 210d = 42$ ---- (ii)

Solving (i) and (ii) simultaneously:

$a + 8d = 3 \Rightarrow a = 3 - 8d$

$21a + 210d = 42$

$\Rightarrow 21(3 - 8d) + 210d = 42$

$63 - 168d + 210d = 42$

$42d = -21$

$d = \frac{-21}{42} = -0.5$

$\therefore d = -0.5$

$a = 3 - 8(-0.5)$

$= 3 + 4 = 7$

$\therefore a = 7$

$\Rightarrow$ first term $(a) = 7$

Common difference $(d) = -0.5$

Do not write outside the box

6 (b) A second arithmetic series has first term -18 and common difference $\frac{3}{4}$

The sum of the first n terms of this series is T_n

Find the value of n such that $T_n = S_n$

[3 marks]

S_n, $a = 7$, $d = -0.5 \Rightarrow S_n = \frac{n}{2}(2a + (n-1)d)$

$S_n = \frac{n}{2}(2(7) - 0.5(n-1))$

$= \frac{n}{2}(14 - 0.5n + 0.5)$

$= \frac{n}{2}(14.5 - 0.5n)$

T_n : $a = -18$, $d = 3/4 = 0.75$

$T_n = \frac{n}{2}(2(-18) + 0.75(n-1))$

$= \frac{n}{2}(-36 + 0.75n - 0.75)$

$= \frac{n}{2}(-36.75 + 0.75n)$

$S_n = T_n$

$\cancel{\frac{n}{2}}(14.5 - 0.5n) = \cancel{\frac{n}{2}}(-36.75 + 0.75n)$

Turn over for the next question

$\Rightarrow 14.5 - 0.5n = -36.75 + 0.75n$

$14.5 + 36.75 = 0.75n + 0.5n$

$\frac{51.25}{1.25} = \frac{\cancel{1.25}n}{\cancel{1.25}}$

$\Rightarrow n = 41$

$\therefore n = 41$

Do not write outside the box

7 The equation $x^2 = x^3 + x - 3$ has a single solution, $x = \alpha$

7 (a) By considering a suitable change of sign, show that α lies between 1.5 and 1.6

[2 marks]

$x^2 = x^3 + x - 3$

$x^3 - x^2 + x - 3 = 0$

Let $f(x) = x^3 - x^2 + x - 3$

$f(1.5) = (1.5)^3 - (1.5)^2 + 1.5 - 3$

$= -0.375 < 0$

$f(1.6) = (1.6)^3 - (1.6)^2 + 1.6 - 3$

$= 0.136 > 0$

Hence α lies between 1.5 and 1.6

7 (b) Show that the equation $x^2 = x^3 + x - 3$ can be rearranged into the form

$$x^2 = x - 1 + \frac{3}{x}$$

[2 marks]

$x^2 = x^3 + x - 3$

$\Rightarrow x^3 = x^2 - x + 3$

Multiplying both sides by $\left(\frac{1}{x}\right)$

$\frac{1}{x}(x^3) = \frac{1}{x}(x^2 - x + 3)$

$\therefore x^2 = x - 1 + \frac{3}{x}$

Do not write outside the box

7 (c) Use the iterative formula

$$x_{n+1} = \sqrt{x_n - 1 + \frac{3}{x_n}}$$

with $x_1 = 1.5$, to find x_2, x_3 and x_4, giving your answers to four decimal places.

[2 marks]

$x_2 = \sqrt{x_1 - 1 + \frac{3}{x_1}}$

$= \sqrt{1.5 - 1 + \frac{3}{1.5}}$

$\therefore x_2 = 1.5811$

$x_3 = \sqrt{x_2 - 1 + \frac{3}{x_2}}$

$= \sqrt{1.5811 - 1 + \frac{3}{1.5811}}$

$\therefore x_3 = 1.5743$

$x_4 = \sqrt{x_3 - 1 + \frac{3}{x_3}}$

$= \sqrt{1.5743 - 1 + \frac{3}{1.5743}}$

$\therefore x_4 = 1.5748$

7 (d) Hence, deduce an interval of width 0.001 in which α lies.

[1 mark]

$x_3 = 1.5743$, $x_4 = 1.5748 \simeq 1.575$

Therefore α lies between

$-1.574 \leq \alpha \leq 1.575$

Turn over for the next question

Do not write outside the box

8 (a) Given that

$$9\sin^2\theta + \sin 2\theta = 8$$

show that

$$8\cot^2\theta - 2\cot\theta - 1 = 0$$

[4 marks]

$9\sin^2\theta + \sin 2\theta = 8$

$9\sin^2\theta + 2\sin\theta\cos\theta = 8$

Multiplying bote sides by $\frac{1}{\sin^2\theta}$

$\frac{1}{\sin^2\theta}(9\sin^2\theta + 2\sin\theta\cos\theta) = \frac{1}{\sin^2\theta}(8)$

$\Rightarrow 9 + \frac{2\cos\theta}{\sin\theta} = \frac{8}{\sin^2\theta}$

But $\frac{\cos\theta}{\sin\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta}} = \frac{1}{\tan\theta} = \cot\theta$

and $\frac{1}{\sin^2\theta} = \operatorname{cosec}^2\theta$

$\Rightarrow 9 + 2\frac{\cos\theta}{\sin\theta} = \frac{8}{\sin^2\theta}$

$\Rightarrow 9 + 2\cot\theta = 8\operatorname{cosec}^2\theta$

$9 + 2\cot\theta = 8(\cot^2\theta + 1)$

$9 + 2\cot\theta = 8\cot^2\theta + 8$

$\Rightarrow 8\cot^2\theta + 8 - 2\cot\theta - 9 = 0$

$8\cot^2\theta - 2\cot\theta - 1 = 0$ As required.

Do not write outside the box

8 (b) Hence, solve

$$9\sin^2\theta + \sin 2\theta = 8$$

in the interval $0 < \theta < 2\pi$

Give your answers to two decimal places.

[3 marks]

$9\sin^2 + \sin 2\theta = 8$

$\Rightarrow 8\cot^2\theta - 2\cot\theta - 1 = 0$

Let $x = \cot\theta$

$8x^2 - 2x - 1 = 0$

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(8)(-1)}}{2(8)}$

$= \dfrac{2 \pm 6}{16}$

$\therefore x = \frac{1}{2}$ or $-\frac{1}{4}$

$\Rightarrow x = \cot\theta = \frac{1}{2} \Rightarrow \tan\theta = 2$

$\cot\theta = -\frac{1}{4} \Rightarrow \tan\theta = -4$

$\tan\theta = 2$ or $\tan\theta = -4$

$\theta = \tan^{-1}2$ or $\theta = \tan^{-1}(-4)$

1.107 1.8158

or   or

$1.107 + \pi = 4.2487$ $1.8158 + \pi = 4.957$

$\therefore \theta = 1.107,\ 1.8158,\ 4.2487,\ 4.957$

8 (c) Solve

$$9\sin^2\left(2x - \frac{\pi}{4}\right) + \sin\left(4x - \frac{\pi}{2}\right) = 8$$

in the interval $0 < x < \frac{\pi}{2}$

Give your answers to one decimal place.

[2 marks]

Comparing the equation with $9\sin^2\theta + \sin 2\theta = 8$

$\Rightarrow \theta = \left(2x - \frac{\pi}{4}\right)$ or $2\theta = \left(4x - \frac{\pi}{4}\right)$

from 8(b) $\theta = 1.107 = 2x - \frac{\pi}{4}$

$\Rightarrow x = \frac{1}{2}\left(1.107 + \frac{\pi}{4}\right) = 0.946$

or

$\theta = 1.8158 \Rightarrow 2x - \frac{\pi}{4} = 1.8158$

$x = \frac{1}{2}\left(1.8158 + \frac{\pi}{4}\right)$

$= 1.3$

$\therefore x = 0.946,\ 1.3$

Do not write outside the box

9 The table below shows the annual global production of plastics, P, measured in millions of tonnes per year, for six selected years.

Year	1980	1985	1990	1995	2000	2005
P	75	94	120	156	206	260

It is thought that P can be modelled by

$$P = A \times 10^{kt}$$

where t is the number of years after 1980 and A and k are constants.

9 (a) Show algebraically that the graph of $\log_{10} P$ against t should be linear.

[3 marks]

$P = A \times 10^{kt}$

Introducing Logarithm on both sides:

$\log_{10} P = \log_{10}(A \times 10^{kt})$

$\log_{10} P = \log_{10} A + \log_{10} 10^{kt}$

$\log_{10} P = \log_{10} A + kt \log_{10} 10$

But $\log_{10} 10 = 1$

$\log_{10} P = \log_{10} A + kt$

9 (b) (i) Complete the table below.

t	0	5	10	15	20	25
$\log_{10} P$	1.88	1.97	2.08	2.19	2.31	2.41

[1 mark]

When $t = 15$

$\log_{10} P = \log_{10} 156 = 2.19$

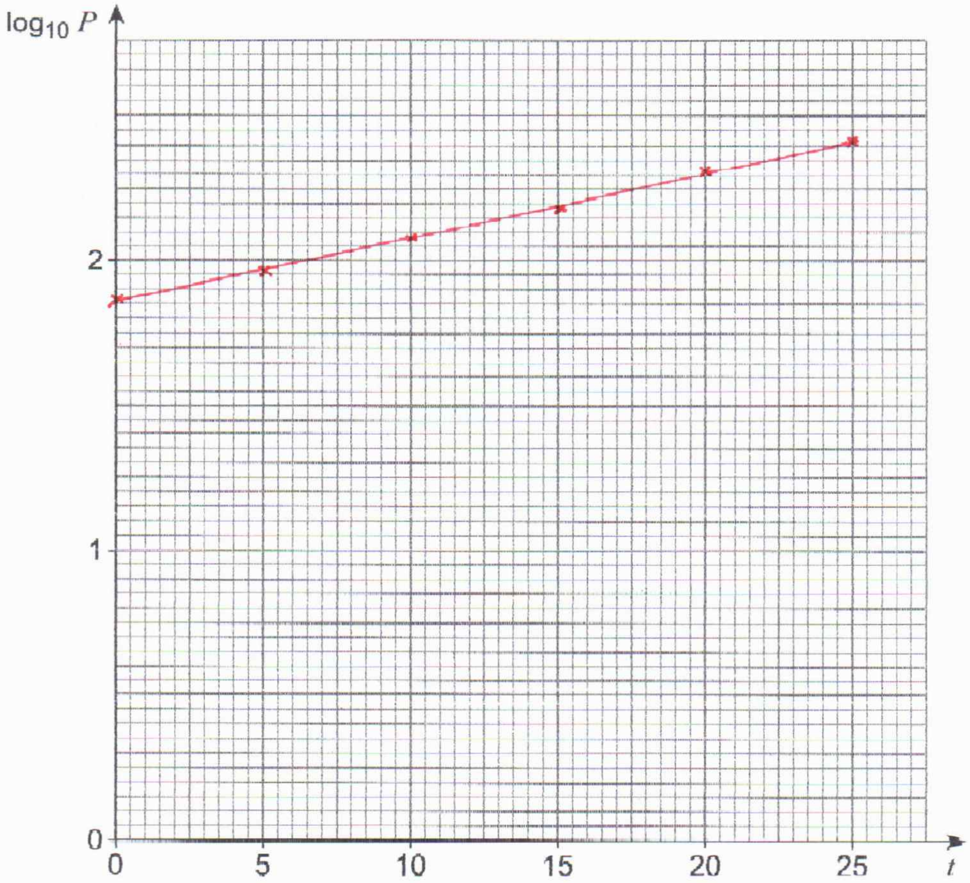
$t = 25$

$\log_{10} P = \log_{10} 260 = 2.41$

Do not write outside the box

9 (b) (ii) Plot $\log_{10} P$ against t, and draw a line of best fit for the data.

[2 marks]



9 (c) (i) Hence, show that k is approximately 0.02

[2 marks]

Taking the first point and the last point.

$k = \dfrac{2.41 - 1.88}{25 - 0}$

$= 0.0212 \simeq 0.02$

$\therefore k = 0.02$

9 (c) (ii) Find the value of A.

[1 mark]

A is the value of the y-intercept $= 1.88$

$\log_{10} P = 1.88$

$\Rightarrow P = 10^{1.88} = 75$

$\therefore A = 75$

9 (d) Using the model with $k = 0.02$ predict the number of tonnes of annual global production of plastics in 2030.

[2 marks]

$P = A \times 10^{kt}$ $A = 75$, $k = 0.02$

$t = (2030 - 1980) = 50$

$P = 75 \times 10^{(0.02 \times 50)}$

$= 75 \times 10^{1}$

$= 750$

$= 750$ million tonnes.

9 (e) Using the model with $k = 0.02$ predict the year in which P first exceeds 8000

[3 marks]

$P = A \times 10^{kt}$

$8000 = 75 \times 10^{0.02t}$

Introducing Logarithm on both sides:

$\log_{10}\left(\frac{8000}{75}\right) = \log 10^{0.02t}$

$\log_{10} 106.6667 = 0.02t \log_{10} 10$

$\frac{2.028}{0.02} = \frac{0.02t}{0.02}$

$\therefore t = 101.4 \simeq 101$

Year $= 1980 + 101 + 1$

$= 2082$

$\therefore$ Year $= 2082$

9 (f) Give a reason why it may be inappropriate to use the model to make predictions about future annual global production of plastics.

[1 mark]

The world will produce less plastics to be more environmentally friendly.

Do not write outside the box

10 (a) Given that

$$y = \tan x$$

use the quotient rule to show that

$$\frac{dy}{dx} = \sec^2 x$$

[3 marks]

$y = \tan x = \dfrac{\sin x}{\cos x}$

$\dfrac{d}{dx}\left(\dfrac{\sin x}{\cos x}\right)$

Using quotient rule $\dfrac{dy}{dx} = \dfrac{vu' - uv'}{v^2}$

let $u = \sin x$ $\quad v = \cos x$

$u' = \cos x$ $\quad v' = -\sin x$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{\cos x(\cos x) - \sin x(-\sin x)}{(\cos x)^2}$

$= \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}$

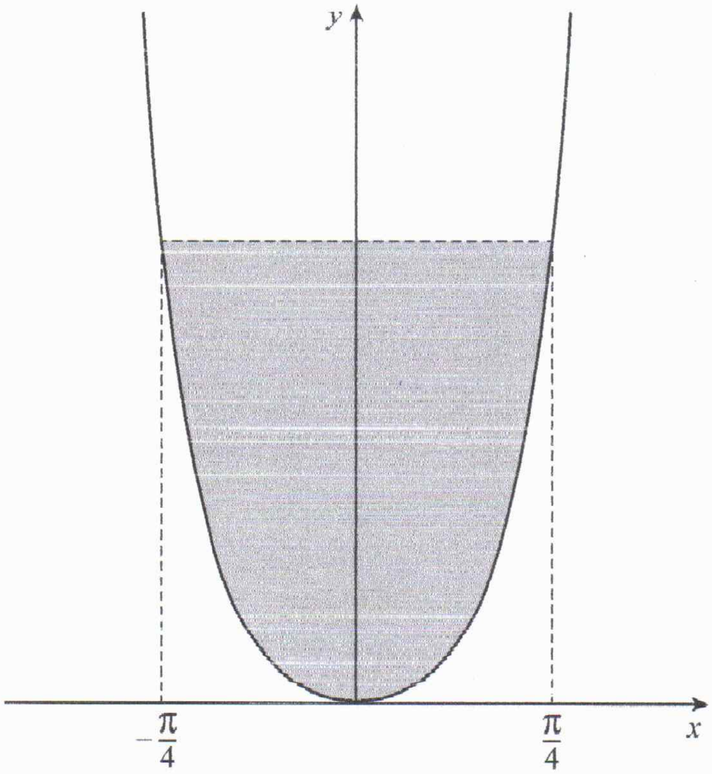
But $\cos^2 x + \sin^2 x = 1$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\cos^2 x} = \sec^2 x$

$\therefore \dfrac{dy}{dx} = \sec^2 x$

Do not write outside the box

10 (b) The region enclosed by the curve $y = \tan^2 x$ and the horizontal line, which intersects the curve at $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$, is shaded in the diagram below.



Show that the area of the shaded region is

$$\pi - 2$$

Fully justify your answer.

[5 marks]

Area under the curve $= \int_a^b y \, dx$

$= \int_{-\pi/4}^{\pi/4} \tan^2 x \, dx$

$= \int_{-\pi/4}^{\pi/4} (\sec^2 x - 1) \, dx$

Do not write outside the box

$$= \Big[\tan x - x \Big]_{-\pi/4}^{\pi/4}$$

$$= \left(\tan\left(\pi/_4\right) - \pi/_4\right) - \left(\tan\left(-\pi/_4\right) - \left(-\pi/_4\right)\right)$$

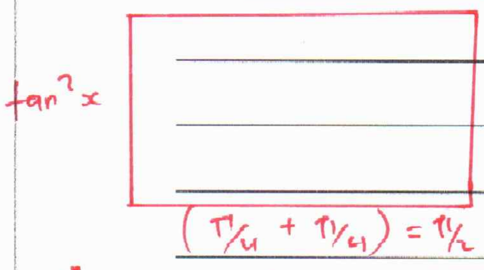
$$= 1 - \pi/_4 - \left(-1 + \pi/_4\right)$$

$$= 1 - \pi/_4 + 1 - \pi/_4$$

$$= 2 - \frac{2\pi}{4}$$

$$= 2 - \frac{\pi}{2}$$

Area of the rectangle



$$\pi/_2 \times \tan^2\left(\pi/_4\right)$$

$$= \frac{\pi}{2}$$

Area of the shaded region = Area of the rectangle − Area under the curve

$$= \frac{\pi}{2} - \left(2 - \pi/_2\right)$$

$$= \frac{\pi}{2} - 2 + \pi/_2$$

$$= \pi - 2$$

Do not write outside the box

11 A curve, C, passes through the point with coordinates (1, 6)

The gradient of C is given by

$$\frac{dy}{dx} = \frac{1}{6}(xy)^2$$

Show that C intersects the coordinate axes at exactly one point and state the coordinates of this point.

Fully justify your answer.

[8 marks]

$$\frac{dy}{dx} = \frac{x^2y^2}{6}$$

$$\int \frac{1}{y^2}\, dy = \int \frac{x^2}{6}\, dx$$

$$= \int y^{-2}\, dy = \frac{1}{6}\int x^2\, dx$$

$$\frac{y^{-1}}{-1} = \frac{1}{6}\left(\frac{x^3}{3}\right) + c$$

$$\frac{-1}{y} = \frac{x^3}{18} + c$$

$y \neq 0$ because y is undefined at $y = 0$, therefore C does not intersect the x-axis.

At (1, 6)

$$\frac{-1}{6} = \frac{1}{18} + c$$

$$\Rightarrow c = \frac{-1}{6} - \frac{1}{18} = -2/9$$

$$\therefore -\frac{1}{y} = \frac{x^3}{18} - \frac{2}{9}$$

$$= \frac{1}{y} = -\frac{x^3}{18} + 2/9$$

When $x = 0$

$$\frac{1}{y} = 0 + 2/9$$

$$\Rightarrow y = \frac{9}{2} = 4.5$$

Therefore the curve C crosses the y-axis at $(0, 4.5)$

Turn over for the next question

12 The equation of a curve is

$$(x+y)^2 = 4y + 2x + 8$$

The curve intersects the positive x-axis at the point P.

12 (a) Show that the gradient of the curve at P is $-\frac{3}{2}$

[6 marks]

If the curve intersects the positive x-axis at point $P \Rightarrow y = 0$

$$(x+0)^2 = 4(0) + 2x + 8$$

$$x^2 = 2x + 8$$

$$x^2 - 2x - 8 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2}, \quad \frac{2 \pm 6}{2}$$

$\therefore$ $x = 4$ or $x = -2$ But $x > 0$

$\Rightarrow$ At P, $x = 4$, $y = 0$

Differentiating $(x+y)^2 = 4y + 2x + 8$ implicitly

$$x^2 + 2xy + y^2 = 4y + 2x + 8$$

$$2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 4\frac{dy}{dx} + 2$$

$$\Rightarrow 2x\frac{dy}{dx} + 2y\frac{dy}{dx} - 4\frac{dy}{dx} = 2 - 2x - 2y$$

$$\frac{dy}{dx}(2x + 2y - 4) = 2 - 2x - 2y$$

$$\frac{dy}{dx} = \frac{2 - 2x - 2y}{2x + 2y - 4}$$

At $4, 0$

$$\frac{dy}{dx} = \frac{2 - 2(4) - 0}{2(4) + 0 - 4}$$

$$= \frac{-6}{4} = \frac{-3}{2}$$

$\therefore$ $\frac{dy}{dx} = \text{gradient} = -\frac{3}{2}$

Do not write outside the box

12 (b) Find the equation of the normal to the curve at P, giving your answer in the form $ax + by = c$, where a, b and c are integers.

[2 marks]

Gradient of of the normal $= \frac{-1}{-3/2} = 2/3$

$y - 0 = +\frac{2}{3}(x - 4)$

$y = +\frac{2}{3}x + \left(-\frac{8}{3}\right)$

$3y = -8 + 2x$

$2x - 3y = 8$

Turn over for the next question

Do not write outside the box

13 (a) Given that

$$P(x) = 125x^3 + 150x^2 + 55x + 6$$

use the factor theorem to prove that $(5x + 1)$ is a factor of $P(x)$.

[2 marks]

$5x + 1 = 0 \Rightarrow x = -1/5$

$P\left(\frac{-1}{5}\right) = 125\left(-1/5\right)^3 + 150\left(-1/5\right)^2 + 55\left(-1/5\right) + 6$

$= -1 + 6 - 11 + 6$

$= 0$

$\Rightarrow$ If $P(-1/5) = 0$, then $(5x+1)$ is a factor of $P(x)$

13 (b) Factorise $P(x)$ completely.

[3 marks]

Using synthetic division :

$$\begin{array}{r|l} & 25x^2 + 25x + 6 \\ \hline 5x+1 & 125x^3 + 150x^2 + 55x + 6 \\ & \underline{125x^3 + 25x^2} \\ & \qquad\qquad 125x^2 + 55x \\ & \qquad\qquad \underline{125x^2 + 25x} \\ & \qquad\qquad\qquad 30x + 6 \\ & \qquad\qquad\qquad \underline{30x + 6} \\ & \qquad\qquad\qquad\qquad 0 \end{array}$$

$\Rightarrow (5x+1)(25x^2 + 25x + 6)$

$(5x+1)((25x^2 + 10x) + (15x + 6))$

$(5x+1)(5x(5x+2) + 3(5x+2))$

$(5x+1)(5x+2)(5x+3)$

$= (5x+1)(5x+2)(5x+3)$

Do not write outside the box

13 (c) Hence, prove that $250n^3 + 300n^2 + 110n + 12$ is a multiple of 12 when n is a positive whole number.

[3 marks]

$250n^3 + 300n^2 + 110n + 12 = 2(5n+1)(5n+2)(5n+3)$

$\Rightarrow$ $(5n+1)$ $(5n+2)$ and $(5n+3)$ is a product of three consecutive whole numbers.

The three algebraic factors must contain a multiple of 3 and must also contain a multiple of 2 and the extra 2 gives:

$2 \times 2 \times 3 = 12$

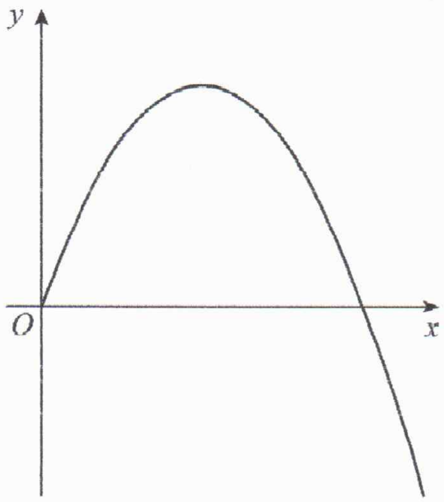
Therefore $250n^3 + 300n^2 + 110n + 12$ is a multiple of 12

Turn over for the next question

14 The curve C is defined for $t \geq 0$ by the parametric equations

$$x = t^2 + t \quad \text{and} \quad y = 4t^2 - t^3$$

C is shown in the diagram below.



14 (a) Find the gradient of C at the point where it intersects the positive x-axis.

[5 marks]

If C intersects the positive x-axis, $y = 0$

$0 = 4t^2 - t^3$

$t^2(4 - t) = 0$

$\Rightarrow t = 0$ or $t = 4$

Gradient $= \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$\frac{dy}{dt} = 8t - 3t^2$

$\frac{dx}{dt} = 2t + 1 \Rightarrow \frac{dt}{dx} = \frac{1}{2t+1}$

$\Rightarrow \frac{dy}{dx} = 8t - 3t^2 \cdot \frac{1}{2t+1}$

$= \frac{8t - 3t^2}{2t+1}$

At $t = 4$

$\frac{dy}{dx} = \frac{8(4) - 3(4^2)}{2(4) + 1} = -\frac{16}{9}$

$\therefore$ Gradient $= \frac{dy}{dx} = -\frac{16}{9}$

Do not write outside the box

14 (b) (i) The area A enclosed between C and the x-axis is given by

$$A = \int_0^b y\,dx$$

Find the value of b.

[1 mark]

When $y = 0$, $t = 0$ or $t = 4$

$x = t^2 + t$

When $t = 0$, $x = 0$

$t = 4$, $x = 4^2 + 4 = 16 + 4 = 20 \Rightarrow 0 \leq x \leq 20$

$\therefore b = 20$

14 (b) (ii) Use the substitution $y = 4t^2 - t^3$ to show that

$$A = \int_0^4 (4t^2 + 7t^3 - 2t^4)\,dt$$

[3 marks]

$x = t^2 + t$

$\frac{dx}{dt} = 2t + 1$

$\Rightarrow dx = (2t+1)\,dt$

$A = \int_0^{20} y\,dx$

$= \int_0^4 (4t^2 - t^3)(2t+1)\,dt$

$= \int_0^4 4t^2(2t+1) - t^3(2t+1)\,dt$

$= \int_0^4 (8t^3 + 4t^2 - 2t^4 - t^3)\,dt$

$= \int_0^4 (7t^3 + 4t^2 - 2t^4)\,dt$

$= \int_0^4 (4t^2 + 7t^3 - 2t^4)\,dt$

14 (b) (iii) Find the value of A.

[1 mark]

$\int_0^4 (4t^2 + 7t^3 - 2t^4)\,dt$

$= \left. \frac{4t^3}{3} + \frac{7t^4}{4} - \frac{2t^5}{5} \right|_0^4$

$= \frac{4}{3}(4^3) + \frac{7}{4}(4^4) - \frac{2}{5}(4^5)$

$= 85\frac{1}{3} + 448 - 409\frac{3}{5}$

$= \frac{1856}{15}$

$\therefore A = \frac{1856}{15}$

$\simeq 123.733$

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Do not write outside the box

15 (a) Show that

$$\sin x - \sin x \cos 2x \approx 2x^3$$

for small values of x.

[3 marks]

Using Small angle approximation

$\sin x \approx x$

$\cos x \approx 1 - \frac{x^2}{2} \Rightarrow \cos 2x \approx 1 - \frac{(2x)^2}{2}$

$\approx 1 - 2x^2$

$\sin x - \sin x \cos 2x \approx x - x(1 - 2x^2)$

$\approx x - x + 2x^3$

$\approx 2x^3$

$\therefore \sin x - \sin x \cos 2x \approx 2x^3$

15 (b) Hence, show that the area between the graph with equation

$$y = \sqrt{8(\sin x - \sin x \cos 2x)}$$

the positive x-axis and the line $x = 0.25$ can be approximated by

$$\text{Area} \approx 2^m \times 5^n$$

where m and n are integers to be found.

[4 marks]

$\text{Area} = \int_0^{0.25} y \, dx$

$= \int_0^{0.25} \sqrt{8(\sin x - \sin x \cos 2x} \, dx$

$= \int_0^{0.25} \sqrt{16x^3} \, dx = \int_0^{0.25} 4x^{3/2} \, dx$

$= 4 \int_0^{0.25} x^{3/2} \, dx$

$= 4 \left[\frac{x^{5/2}}{5/2} \right]_0^{0.25}$

Do not write outside the box

$= 4\left[\left(\frac{2x^{5/2}}{5}\right)\Big|_0^{0.25}\right]$

$= 4\left(\frac{2(0.25)^{5/2}}{5}\right)$

$= 4\left(\frac{2 \times (1/2)^5}{5}\right)$

$= \frac{8}{5} \times (1/2)^5 \Rightarrow$ Recall $(1/2)^x = 2^{-x}$

$= \frac{1}{5} \times 8 \times 2^{-5}$

$= 5^{-1} \times 2^3 \times 2^{-5}$

$= 2^{-2} \times 5^{-1}$

$\therefore$ Area $= 2^{-2} \times 5^{-1}$

15 (c) (i) Explain why

$$\int_{6.3}^{6.4} 2x^3 \, dx$$

is **not** a suitable approximation for

$$\int_{6.3}^{6.4} (\sin x - \sin x \cos 2x) \, dx$$

[1 mark]

6.3 and 6.4 are not small values of x and the approximation is only valid for small values.

Question 15 continues on the next page

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15 (c) (ii) Explain how

$$\int_{6.3}^{6.4} (\sin x - \sin x \cos 2x)\, \mathrm{d}x$$

may be approximated by

$$\int_a^b 2x^3\, \mathrm{d}x$$

for suitable values of a and b.

[2 marks]

Since (Sinx - Sinx cos 2x) is repetitive, we evaluate the integral over a different value using Small values.

Let $a = 6.3 - 2\pi$
$b = 6.4 - 2\pi$ To obtain a valid approximation.

END OF QUESTIONS