AQA

Please write clearly in block capitals.

Centre number						Candidate number				

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

A-level
MATHEMATICS
Paper 2

Wednesday 12 June 2019 Morning Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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PB/Jun19/E4

7357/2

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Section A

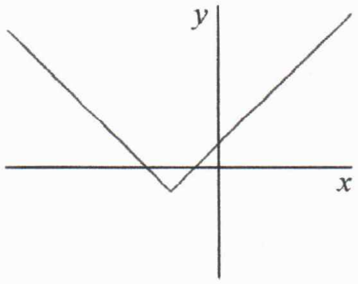
Answer **all** questions in the spaces provided.

1 Identify the graph of $y = 1 - |x + 2|$ from the options below.

Tick (✓) **one** box.

[1 mark]

A



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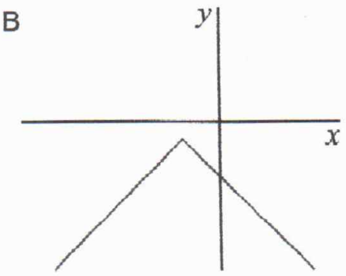

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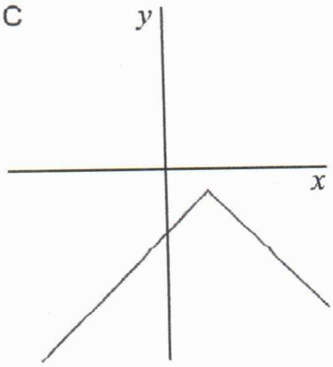

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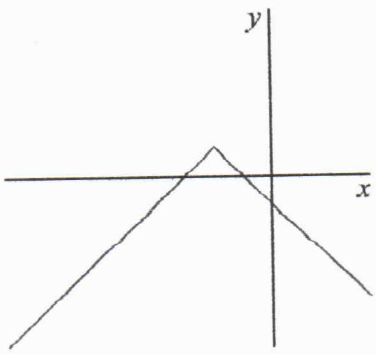

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2 Simplify $\sqrt{a^{\frac{2}{3}} \times a^{\frac{2}{5}}}$

Circle your answer.

From the laws of indices
$a^b \times a^c = a^{b+c}$
$(a^b)^2 = a^{2b}$

[1 mark]

$a^{\frac{2}{15}}$ $a^{\frac{4}{15}}$ $a^{\frac{8}{15}}$ $a^{\frac{16}{15}}$

$a^{2/3} \times a^{2/5} = a^{(2/3 + 2/5)} = a^{16/15}$
$(a^{16/15})^{1/2} = a^{16/15 \times 1/2} = a^{8/15}$.

3 Each of these functions has domain $x \in \mathbb{R}$

Which function does **not** have an inverse?

Circle your answer.

[1 mark]

$f(x) = x^3$ $f(x) = 2x + 1$ $f(x) = x^2$ $f(x) = e^x$

A function to have an inverse it must be one to one, that is for each value of y, there is only one possible value of x.

$f(x) = x^2$ is not one to one

example $f(1) = 1^2$
$f(-1) = 1^2$

Turn over for the next question

Do not write outside the box

4 $x^2 + bx + c$ and $x^2 + dx + e$ have a common factor $(x + 2)$

Show that $2(d - b) = e - c$

Fully justify your answer.

[4 marks]

If $(x+2)$ is a factor, when $x = -2$, $f(x) = 0$.

When $x = -2$, $x^2 + bx + c = 0$

$(-2)^2 + b(-2) + c = 0$

$4 - 2b + c = 0$ .... (i)

When $x = -2$ $x^2 + dx + e = 0$

$(-2)^2 + d(-2) + e = 0$

$4 - 2d + e = 0$ ....(ii)

Equating equation (i) and (ii)

$4 - 2b + c = 4 - 2d + e$

$4 - 4 - 2b + 2d = e - c$

$-2b + 2d = e - c$

$2(-b + d) = e - c$

$= 2(d - b) = e - c$

5 Solve the differential equation

$$\frac{dt}{dx} = \frac{\ln x}{x^2 t} \qquad \text{for } x > 0$$

given $x = 1$ when $t = 2$

Write your answer in the form $t^2 = f(x)$

[7 marks]

$$\frac{dt}{dx} = \frac{\ln x}{x^2 t}$$

$$\frac{\ln x}{x^2}\,dx = \frac{\cancel{x^2} t}{\cancel{x^2}}\,dt$$

$$\int \frac{\ln x}{x^2}\,dx = \int t\,dt$$

To solve $\int \frac{\ln x}{x^2}\,dx$ we use integration by parts

$$\int u\,dv = uv - \int v\,du$$

Let $u = \ln x$

$du = \frac{1}{x}$

Let $dv = \frac{1}{x^2} = x^{-2}$

$v = \int x^{-2}\,dx$

$v = -x^{-1} = -\frac{1}{x}$

$$\int \frac{\ln x}{x^2} = \frac{-\ln x}{x} - \int \frac{-1}{x} \cdot \frac{1}{x}\,dx$$

$$= \frac{-\ln x}{x} - \int \frac{-1}{x^2}\,dx$$

$$= \frac{-\ln x}{x} + \int \frac{1}{x^2}\,dx$$

$$= \frac{-\ln x}{x} - \frac{1}{x}$$

$$\Rightarrow \int \frac{\ln x}{x}\,dx = \int t\,dt$$

$$= \frac{-\ln x}{x} - \frac{1}{x} = \frac{t^2}{2} + c$$

When $x = 1$, $t = 2$

$$\Rightarrow -1(\ln(1)) - 1 = \frac{(2)^2}{2} + c$$

$$-1 = 2 + c$$

$$\Rightarrow c = -1 - 2 = -3$$

$$\frac{-\ln x}{x} - \frac{1}{x} = \frac{t^2}{2} - 3$$

Multiplying by 2

$$\Rightarrow t^2 - 6 = 2\left(\frac{-\ln x}{x} - \frac{1}{x}\right)$$

$$t^2 = 6 + 2\left(-\frac{1}{x}\ln x - \frac{1}{x}\right)$$

$$= 6 - 2\left(\frac{\ln x + 1}{x}\right)$$

$$\therefore t^2 = 6 - 2\left(\frac{\ln x + 1}{x}\right)$$

Turn over for the next question

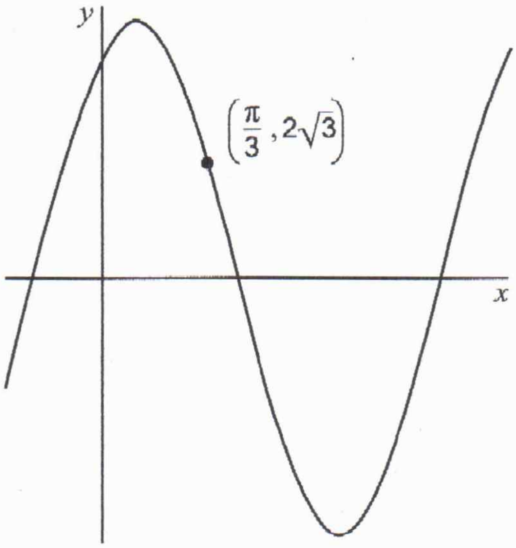
Do not write outside the box

6 A curve has equation

$$y = a\sin x + b\cos x$$

where a and b are constants.

The maximum value of y is 4 and the curve passes through the point $\left(\frac{\pi}{3}, 2\sqrt{3}\right)$ as shown in the diagram.



Find the exact values of a and b.

[6 marks]

$y = a\sin x + b\cos x$

$R\sin(x + \alpha) = a\sin x + b\cos x$

Maximum value of $y = 4 = R$

$\Rightarrow 4\sin(x + \alpha) = a\sin x + b\cos x$

At the point $\left(\pi/3, 2\sqrt{3}\right)$

$4\sin\left(\pi/3 + \alpha\right) = 2\sqrt{3}$

$\sin\left(\pi/3 + \alpha\right) = \frac{2\sqrt{3}}{4}$

$\sin\left(\pi/3 + \alpha\right) = \sqrt{3}/2$

$\pi/3 + \alpha = \sin^{-1}\left(-\sqrt{3}/2\right)$

$\pi/3 + \alpha = 2\pi/3$

Do not write outside the box

$\Rightarrow \quad \alpha = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}.$

Recall $\quad R \sin \alpha = b$

$R \cos \alpha = a.$

$4 \sin \frac{\pi}{3} = b$

$4 \times \frac{\sqrt{3}}{2} = b$

$2\sqrt{3} = b$

Turn over for the next question

$R \cos \alpha = a$

$4 \cos \frac{\pi}{3} = a$

$4 \times \frac{1}{2} = a$

$\Rightarrow \quad a = 2$

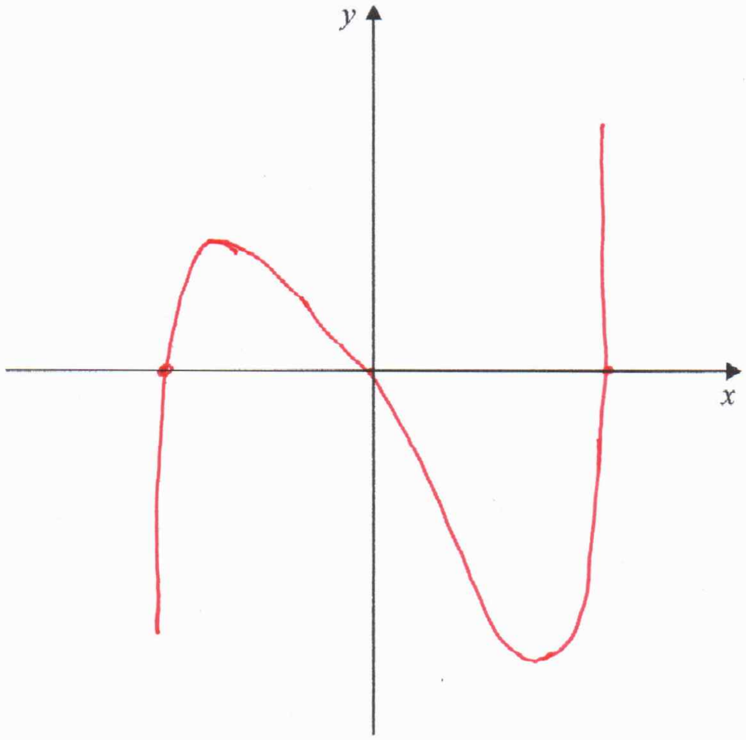
$\therefore \quad a = 2$

$b = 2\sqrt{3}$

Turn over ▶

7 (a) Sketch the graph of any cubic function that has **both** three distinct real roots **and** a positive coefficient of x^3

[2 marks]



7 (b) The function $f(x)$ is defined by

$$f(x) = x^3 + 3px^2 + q$$

where p and q are constants and $p > 0$

7 (b) (i) Show that there is a turning point where the curve crosses the y-axis.

[3 marks]

A turning point of a function is a point where $f'(x) = 0$

$f(x) = x^3 + 3px^2 + q$

$f'(x) = 3x^2 + 6px = 0$

$3x(x + 2p) = 0$

$3x = 0 \Rightarrow x = 0$ or $x + 2p = 0 \Rightarrow x = -2p$

$\Rightarrow$ There are two turning points of the function $f(x)$, because $x = 0$ is a turning point of the function, then there must be a turning point where the curve crosses the y-axis.

Do not write outside the box

7 (b) (ii) The equation $f(x) = 0$ has three distinct real roots.

By considering the positions of the turning points find, in terms of p, the range of possible values of q.

[5 marks]

$x = 0$ is the minimum turning point.

Since $p > 0$, $x = -2p$ is the maximum turning point.

$f(x) = x^3 + 3px^2 + q$ ...... (i)

Substituting $x = 0$ and $x = -2p$ into (i)

$f(0) = 0^3 + 3p(0) + q$

$\Rightarrow f(0) = q$

$f(-2p) = (-2p)^3 + 3p(-2p)^2 + q$

$= -8p^3 + 12p^3 + q$

$\therefore f(-2p) = 4p^3 + q$

Since $p > 0 \Rightarrow -4p^3 < q$

Therefore $-4p^3 < q < 0$

Turn over for the next question

Do not write outside the box

8 Theresa bought a house on 2 January 1970 for £8000.

The house was valued by a local estate agent on the same date every 10 years up to 2010.

The valuations are shown in the following table.

Year	1970	1980	1990	2000	2010
Valuation price	£8 000	£19 000	£36 000	£82 000	£205 000

The valuation price of the house can be modelled by the equation

$$V = pq^t$$

where V pounds is the valuation price t years after 2 January 1970 and p and q are constants.

8 (a) Show that $V = pq^t$ can be written as $\log_{10} V = \log_{10} p + t\log_{10} q$

[2 marks]

$V = pq^t$

Introducing Logarithm on both sides;

$\log_{10} V = \log_{10}(pq^t)$

From the Law of Logarithms;

$\log_{10} pq = \log_{10} p + \log_{10} q$

$\log_{10} q^t = t\log_{10} q$

$\Rightarrow \log_{10} V = \log_{10}(pq^t) = \log_{10} p + \log_{10} q^t$

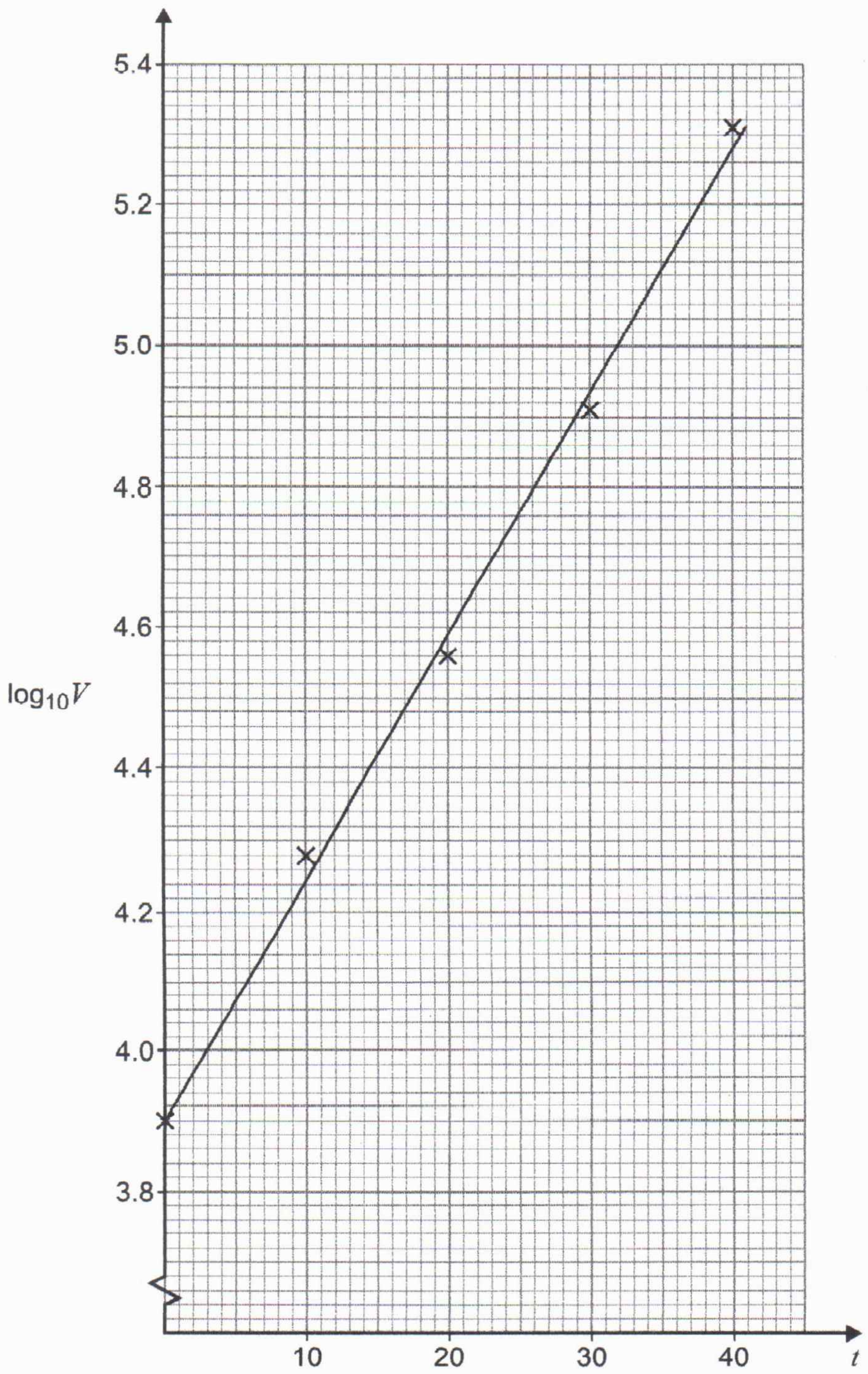
$= \log_{10} p + t\log_{10} q$

$\therefore \log_{10} V = \log_{10} p + t\log_{10} q$ as

required.

8 (b) The values in the table of $\log_{10} V$ against t have been plotted and a line of best fit has been drawn on the graph below.

t	0	10	20	30	40
$\log_{10} V$	3.90	4.28	4.56	4.91	5.31



Question 8 continues on the next page

Turn over ▶

Do not write outside the box

Using the given line of best fit, find estimates for the values of p and q.

Give your answers correct to three significant figures.

[4 marks]

Equation of the line of best fit is of ~~the~~ the form $y = mx + b$ where m is the gradient of the line and b is the y-intercept.

$$\log_{10} v = t \log_{10} q + \log_{10} p$$

Gradient $= \log_{10} q$

Taking two points along the line;

$(10, 4.24)$, $(35, 5.1)$

$$\log_{10} q = \frac{5.1 - 4.24}{35 - 10}$$

$$\log_{10} q = 0.0344$$

$$\Rightarrow q = 10^{(0.0344)} = 1.082679 \simeq 1.08$$

$$\therefore q = 1.08$$

$\log_{10} p$ is the y-intercept $= 3.90$

$$\log_{10} p = 3.90 \Rightarrow p = 10^{(3.90)} = 7943.$$

$$= 7940 \simeq (3 \text{ sf})$$

$$\therefore p = 7940$$

$$q = 1.08$$

Do not write outside the box

8 (c) Determine the year in which Theresa's house will first be worth half a million pounds.

[3 marks]

$V = 500{,}000$

$\log_{10} V = t \log_{10} Q + \log_{10} P$

$V = pq^t$

$500{,}000 = 7940 \times 1.08^t$

$\frac{500{,}000}{7940} = 1.08^t$

$t = \frac{\log 62.9723}{\log 1.08}$

$t = 53.82$

$1970 + 53 = 2023$

$\Rightarrow$ The house will first be worth half a million pounds in 2023.

8 (d) Explain whether your answer to part **(c)** is likely to be reliable.

[2 marks]

· No because the model is only based on data from 1970 to 2010.

· House prices may not continue to grow in the same way indefinitely.

Turn over for the next question

Do not write outside the box

9 (a) Show that the first two terms of the binomial expansion of $\sqrt{4-2x^2}$ are

$$2 - \frac{x^2}{2}$$

[2 marks]

$\sqrt{4-2x^2} = (4-2x^2)^{1/2}$

$= \left(4\left(1-\frac{x^2}{2}\right)\right)^{1/2}$

$\sqrt{4-2x^2} = 2\left(1-\frac{x^2}{2}\right)^{1/2}$

$= 2\left(\binom{1/2}{0} 1^{1/2}\left(\frac{-x^2}{2}\right)^0 + \binom{1/2}{1} 1^{-1/2}\left(\frac{-x^2}{2}\right)^1\right)$

$= 2\left(1 + \frac{1}{2}\left(\frac{-x^2}{2}\right)\right) = 2\left(1-\frac{x^2}{4}\right)$

$= 2 - \frac{x^2}{2}$

9 (b) State the range of values of x for which the expansion found in part **(a)** is valid.

[2 marks]

The range of values of x for $2-\frac{x^2}{2}$ is ~~valid when~~

$\left|-\frac{x^2}{2}\right| < 1$

$\Rightarrow |x^2| < 2$

$\Rightarrow |x| < \sqrt{2}$

Do not write outside the box

9 (c) Hence, find an approximation for

$$\int_0^{0.4} \sqrt{\cos x}\, dx$$

giving your answer to five decimal places.

Fully justify your answer.

[4 marks]

Because 0.4 is a small angle, $\cos x$ will be approximated as $\cos x = 1 - \frac{x^2}{2}$

$$\int_0^{0.4} \sqrt{1 - \frac{x^2}{2}}\, dx$$

$$= \int_0^{4} \left(1 - \frac{x^2}{2}\right)^{1/2} dx$$

Expansion of $\left(1 - \frac{x^2}{2}\right)^{1/2}$

$$= \binom{1/2}{0} 1^{1/2} \left(-\frac{x^2}{2}\right)^0 + \binom{1/2}{1} 1^{-1/2} \left(-\frac{x^2}{2}\right)^1$$

$$= 1 + \frac{1}{2}\left(-\frac{x^2}{2}\right)$$

$$= 1 - \frac{x^2}{4}$$

$$\int_0^{0.4} 1 - \frac{x^2}{4}\, dx$$

$$= x - \frac{x^3}{12}\Big|_0^{0.4}$$

$$= \left(0.4 - \frac{0.4^3}{12}\right) - 0$$

$$= 0.39467$$

9 (d) A student decides to use this method to find an approximation for

$$\int_0^{1.4} \sqrt{\cos x}\, dx$$

Explain why this may not be a suitable method.

[1 mark]

1.4 radians is not a small angle therefore the approximation method is not suitable.

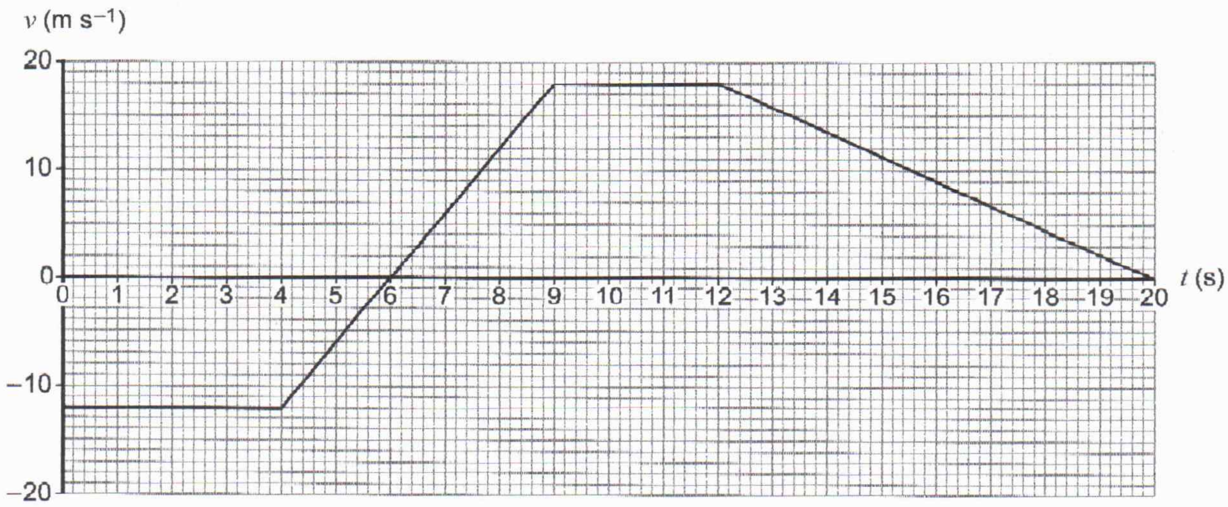
Turn over for Section B

Turn over ▶

Section B

Answer **all** questions in the spaces provided.

10 The diagram below shows a velocity-time graph for a particle moving with velocity v m s^{-1} at time t seconds.



Which statement is correct?

Tick (✓) **one** box.

[1 mark]

- [ ] The particle was stationary for $9 \le t \le 12$
- [x] The particle was decelerating for $12 \le t \le 20$
- [ ] The particle had a displacement of zero when $t = 6$
- [ ] The particle's speed when $t = 4$ was -12 m s^{-1}

$0 \le t \le 4$, the particle was moving with a constant speed

$4 \le t \le 9$, the particle was accelerating

$9 \le t \le 12$, the particle was moving with a constant speed

$12 \le t \le 20$, the particle was decelerating.

Do not write outside the box

11 A wooden crate rests on a rough horizontal surface.

The coefficient of friction between the crate and the surface is 0.6

A forward force acts on the crate, parallel to the surface.

When this force is 600 N, the crate is on the point of moving.

Find the weight of the crate.

Circle your answer.

[1 mark]

1000 N   100 kg   360 N   36 kg

$F = \mu R$, μ = coefficient of friction

$\frac{600}{0.6} = \frac{0.6 R}{0.6} \Rightarrow R = 1000\ N$

12 A particle, under the action of two constant forces, is moving across a perfectly smooth horizontal surface at a constant speed of $10\ \mathrm{m\,s^{-1}}$

The first force acting on the particle is $(400\mathbf{i} + 180\mathbf{j})$ N.

The second force acting on the particle is $(p\mathbf{i} - 180\mathbf{j})$ N.

Find the value of p.

Circle your answer.

[1 mark]

−400   −390   390   400

The second force is acting in the opposite direction.

Turn over for the next question

Do not write outside the box

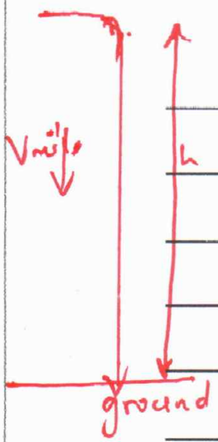
13 In a school experiment, a particle, of mass m kilograms, is released from rest at a point h metres above the ground.

At the instant it reaches the ground, the particle has velocity $v\,\text{m s}^{-1}$

13 (a) Show that

$$v = \sqrt{2gh}$$

[2 marks]



At rest initial velocity $(u) = 0\ \text{ms}^{-1}$

Using equation of motion $v^2 = u^2 + 2as$

$u = 0,\ a = g,\ s = h$

$v^2 = 0 + 2gh$

$v = \sqrt{2gh}$. As required.

13 (b) A student correctly used $h = 18$ and measured v as 20

The student's teacher claims that the machine measuring the velocity must have been faulty.

Determine if the teacher's claim is correct.

Fully justify your answer.

[3 marks]

$v = \sqrt{2gh}$ $\quad g = 9.8,\ h = 18$

$= \sqrt{2 \times 9.8 \times 18}$

$= \sqrt{352.8}$

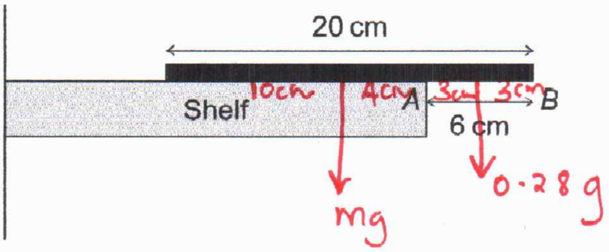
$\therefore v = 18.78$

$18.78 \neq 20$, therefore the teacher's claim is correct that the machine measuring the velocity must have been faulty.

Do not write outside the box

14 A metal rod, of mass m kilograms and length 20 cm, lies at rest on a horizontal shelf.

The end of the rod, B, extends 6 cm beyond the edge of the shelf, A, as shown in the diagram below.



14 (a) The rod is in equilibrium when an object of mass 0.28 kilograms hangs from the midpoint of AB.

Show that $m = 0.21$

[3 marks]

Taking moments about A — converting cm to m

$mg \times \frac{4}{100} = 0.28g \times 3/100$

$mg \times 0.04 = 0.28g \times 0.03$

$0.04m\cancel{g} = 0.0084\cancel{g}$

$m = \frac{0.0084}{0.04}$

$= 0.21$

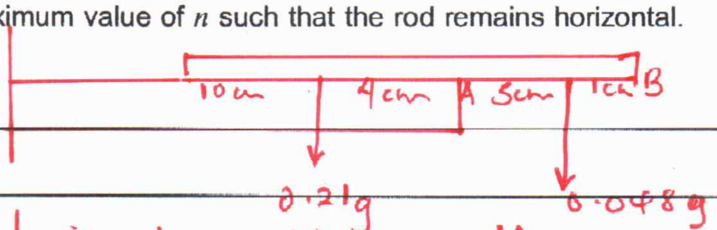
$\therefore m = 0.21$

14 (b) The object of mass 0.28 kilograms is removed.

A number, n, of identical objects, each of mass 0.048 kg, are hung from the rod all at a distance of 1 cm from B.

Find the maximum value of n such that the rod remains horizontal.

[4 marks]



The rod is in equilibrium. Therefore,

Taking moments about A

$0.21g \times \frac{4}{100} = 0.048g \times \frac{5}{100} \times n$

$0.21g \times 0.04 = 0.048g \times 0.05 \times n$

$0.0084\cancel{g} = 0.0024\cancel{g}\, n$

$\Rightarrow n = \frac{0.0084}{0.0024}$

$n = 3.5$

Therefore the maximum value of n is, n = 3

14 (c) State one assumption you have made about the rod.

[1 mark]

The rod is uniform.

The weight of the rod acts in the middle at 10 cm.

Turn over for the next question

Do not write outside the box

15 Four buoys on the surface of a large, calm lake are located at A, B, C and D with position vectors given by

$$\overrightarrow{OA} = \begin{bmatrix} 410 \\ 710 \end{bmatrix}, \overrightarrow{OB} = \begin{bmatrix} -210 \\ 530 \end{bmatrix}, \overrightarrow{OC} = \begin{bmatrix} -340 \\ -310 \end{bmatrix} \text{ and } \overrightarrow{OD} = \begin{bmatrix} 590 \\ -40 \end{bmatrix}$$

All values are in metres.

15 (a) Prove that the quadrilateral $ABCD$ is a trapezium but **not** a parallelogram.

[5 marks]

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\begin{bmatrix} -210 \\ 530 \end{bmatrix} - \begin{bmatrix} 410 \\ 70 \end{bmatrix}$

$\therefore \overrightarrow{AB} = \begin{bmatrix} -620 \\ -180 \end{bmatrix}$

$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$

$= \begin{bmatrix} 590 \\ -40 \end{bmatrix} - \begin{bmatrix} -340 \\ -310 \end{bmatrix}$

$= \begin{bmatrix} 930 \\ 270 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 930 \\ 270 \end{bmatrix} = -1.5 \begin{bmatrix} -620 \\ -180 \end{bmatrix}$

Therefore $\overrightarrow{CD} = -1.5\overrightarrow{AB}$

This implies that CD and AB are parallel but not equal in length, therefore ABCD is a trapezium not a parallelogram.

Do not write outside the box

15 (b) A speed boat travels directly from B to C at a constant speed in 50 seconds.

Find the speed of the boat between B and C.

[4 marks]

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\vec{BC} = \vec{OC} - \vec{OB}$$

$$= \begin{bmatrix} -340 \\ -310 \end{bmatrix} - \begin{bmatrix} -210 \\ 530 \end{bmatrix}$$

$$= \begin{bmatrix} -130 \\ -840 \end{bmatrix}$$

$$|BC| = \sqrt{(-130)^2 + (-840)^2}$$

$$= \sqrt{722500}$$

$$= 850$$

$$\text{Speed} = \frac{850}{50} = 17$$

$$\therefore \text{Speed} = 17\ ms^{-1}$$

Turn over for the next question

Do not write outside the box

16 An elite athlete runs in a straight line to complete a 100-metre race.

During the race, the athlete's velocity, $v\,\text{m s}^{-1}$, may be modelled by

$$v = 11.71 - 11.68e^{-0.9t} - 0.03e^{0.3t}$$

where t is the time, in seconds, after the starting pistol is fired.

16 (a) Find the maximum value of v, giving your answer to one decimal place.

Fully justify your answer.

[8 marks]

Maximum Velocity occurs when acceleration is equal to zero.

$a = \frac{dv}{dt} = 0.$

$V = 11.71 - 11.68e^{-0.9t} - 0.03e^{0.3t}$

$\frac{dv}{dt} = (-11.68 \times -0.9)e^{-0.9t} - (0.03 \times 0.3)e^{0.3t}$

$= 10.512e^{-0.9t} - 0.009e^{0.3t} = 0$

$10.512e^{-0.9t} = 0.009e^{0.3t}$

$\Rightarrow \frac{e^{0.3t}}{e^{-0.9t}} = \frac{10.512}{0.009}$

$e^{(0.3t-(-0.9t))} = 1168$

$e^{1.2t} = 1168$

Introducing Logarithm on both sides;

$\ln e^{1.2t} = \ln(1168)$

Recall $\ln(e^a) = a$

$\Rightarrow 1.2t = 7.063$

$\Rightarrow t = \frac{7.063}{1.2}$

$\therefore t = 5.886$

Replacing t in the equation of V

$V = 11.71 - 11.68e^{(-0.9 \times 5.886)} - 0.03e^{(0.3 \times 5.886)}$

$= 11.71 - 11.68e^{-5.2974} - 0.03e^{1.7658}$

$V = 11.476 \simeq 11.5$

Therefore the maximum value of V when $\frac{dv}{dt} = 0$ is $V = 11.5 \text{ m s}^{-1}$

16 (b) Find an expression for the distance run in terms of t.

[6 marks]

Let the distance be S

$S = \int v \, dt$

$= \int 11.71 - 11.68e^{-0.9t} - 0.03e^{0.3t} \, dt$

$= 11.71t - \frac{11.68}{-0.9}e^{-0.9t} - \frac{0.03}{0.3}e^{-0.3t} + c$

$= 11.71t + 12.978e^{-0.9t} - 0.1e^{-0.3t} + c$

When the athlete is at rest, $t = 0$ and $S = 0$

$\Rightarrow 0 = 0 + 12.978 - 0.1 + c$

$0 = 12.878 + c$

$\Rightarrow c = -12.878$

$\therefore$ Distance $(S) = 11.71t + 12.978e^{-0.9t} - 0.1e^{-0.3t} - 12.878$

Question 16 continues on the next page

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16 (c) The athlete's actual time for this race is 9.8 seconds.

Comment on the accuracy of the model.

[2 marks]

$s = 11.71t + 12.978e^{-0.9t} - 0.1e^{0.3t} - 12.878$

At $t = 9.8$

$s = 11.71(9.8) + 12.978e^{(-0.9\times9.8)} - 0.1e^{(0.3\times9.8)} - 12.878$

$= 114.758 + 12.978e^{-8.82} - 0.1e^{2.94} - 12.878$

$= 99.99m \simeq 100$

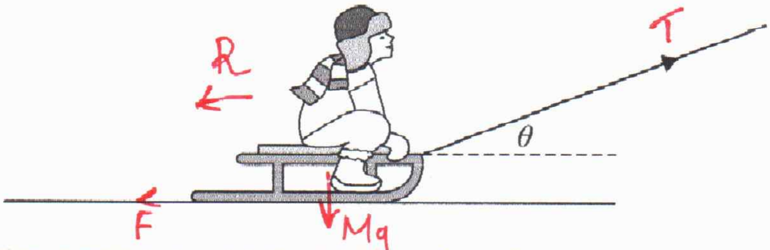
The model predicts the distance to be 99.99m which is very close to 100m, therefore the model is accurate.

17 Lizzie is sat securely on a wooden sledge.

The combined mass of Lizzie and the sledge is M kilograms.

The sledge is being pulled forward in a straight line along a horizontal surface by means of a light inextensible rope, which is attached to the front of the sledge.

This rope stays inclined at an acute angle θ above the horizontal and remains taut as the sledge moves forward.



The sledge remains in contact with the surface throughout.

The coefficient of friction between the sledge and the surface is μ and there are no other resistance forces.

Lizzie and the sledge move forward with constant acceleration, $a\,\mathrm{m\,s^{-2}}$

The tension in the rope is a constant T Newtons.

17 (a) Show that

$$T = \frac{M(a + \mu g)}{\cos\theta + \mu\sin\theta}$$

[7 marks]

Resolving Vertically :

$R(\uparrow)$ $R + T\sin\theta = Mg$

$R = Mg - T\sin\theta$ .... (i)

Resolving horizontally using equation of motion

$F = ma$;

$R(\rightarrow)$ $T\cos\theta - F = Ma$ .... (ii)

Because Lizzie is in limiting equilibrium

$F = \mu R$, where μ is the coefficient of friction.

Replacing the value of F in (ii) with μR ;

$T\cos\theta - \mu R = Ma$ ...... (iii)

Replacing Equation (i) in (iii) gives ;

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$T\cos\theta - \mu(Mg - T\sin\theta) = Ma$

$T\cos\theta - \mu Mg + \mu T\sin\theta = Ma$

$T\cos\theta + \mu T\sin\theta = Ma + \mu Mg$

$\dfrac{T(\cos\theta + \mu\sin\theta)}{\cos\theta + \mu\sin\theta} = \dfrac{M(a + \mu g)}{\cos\theta + \mu\sin\theta}$

$\therefore\ T = \dfrac{M(a + \mu g)}{\cos\theta + \mu\sin\theta}$ As required

17 (b) It is known that when $M = 30$, $\theta = 30°$, and $T = 40$, the sledge remains at rest.

Lizzie uses these values with the relationship formed in part **(a)** to find the value for μ

Explain why her value for μ may be incorrect.

[2 marks]

Friction may not be acting at its limiting value because the sledge remains at rest, hence the relation in part (a) above may not be valid.

END OF QUESTIONS